# The First Law of Thermodynamics and Energy Transport Mechanisms

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# 4.1 INTRODUCCIÓN (INTRODUCTION)

In this chapter, we begin the formal study of the first law of thermodynamics. The theory is presented first, and in subsequent chapters, it is applied to a variety of closed and open systems of engineering interest. In Chapter 4, the first law of thermodynamics and its associated energy balance are developed along with a detailed discussion of the energy transport mechanisms of work and heat. To understand the usefulness of the first law of thermodynamics, we need to study the energy transport modes and investigate the energy conversion efficiency of common technologies.

In Chapter 5, the focus is on applying the theory presented in Chapter 4 to a series of steady state closed systems, such as sealed, rigid containers; electrical apparatuses; and piston-cylinder devices. Chapter 5 ends with a brief discussion of the behavior of unsteady state closed systems.

The first law of thermodynamics is expanded in Chapter 6 to cover open systems, and the conservation of mass law is introduced as a second independent basic equation. Then, appropriate applications are presented, dealing with a variety of common open system technologies of engineering interest, such as nozzles, diffusers, throttling devices, heat exchangers, and work-producing or work-absorbing machines. Chapter 6 ends with a brief discussion of the behavior of unsteady state open systems.

# 4.2 EMMY NOETHER AND THE CONSERVATION LAWS OF PHYSICS

Throughout the long history of physics and engineering, we believed that the conservation laws of momentum, energy, and electric charge were unique laws of nature that had to be discovered and verified by physical experiments. And, in fact, these laws were discovered in this way. They are the heart and soul of mechanics, thermodynamics, and electronics, because they deal with things (momentum, energy, charge) that cannot be created nor destroyed and therefore are "conserved." These conservation laws have broad application in engineering and physics and are considered to be the most fundamental laws in nature.

We have never been able explain where these laws came from because they seem to have no logical source. They seemed to be part of the mystery that is nature. However, almost 100 years ago, the mathematician Emmy Noether developed a theorem that uncovered their source,<sup>1</sup> yet few seem to know of its existence. Emmy Noether's theorem is fairly simple. It states that:

For every symmetry exhibited by a system, there is a corresponding observable quantity that is conserved.

The meaning of the word *symmetry* here is probably not what you think it is. The symmetry that everybody thinks of is called *bilateral* symmetry, when two halves of a whole are each other's mirror images (bilateral symmetry is also called *mirror* symmetry). For example, a butterfly has bilateral symmetry. Emmy Noether was talking about symmetry with respect to a mathematical operation. We say that something has *mathematical* symmetry if, when you perform some mathematical operation on it, it does not change in any way. For example, everyone knows that the equations of physics remain the same under a translation of the coordinate system. This really says that there are no absolute positions in space. What matters is not where an object is in absolute terms, but where it is relative to other objects, that is, its coordinate differences.

The impact of Emmy Noether's studies on symmetry and the behavior of the physical world is nothing less than astounding. Virtually every theory, including relativity and quantum physics, is based on symmetry principles. To quote just one expert, Dr. Lee Smolin, of the Perimeter Institute for Theoretical Physics, "The connection between symmetries and conservation laws is one of the great discoveries of twentieth century physics. But very few non-experts will have heard either of it or its maker—Emily Noether, a great German mathematician. But it is as essential to twentieth century physics as famous ideas like the impossibility of exceeding the speed of light."<sup>2</sup>

Noether's theorem proving that symmetries imply conservation laws has been called the most important theorem in engineering and physics since the Pythagorean theorem. These symmetries define the limit of all possible conservation laws. Is it possible that, had Emmy Noether been a man, all the conservation laws of physics would be called Noether's laws?

<sup>&</sup>lt;sup>1</sup> Noether, E., 1918. Invariante variationsprobleme. *Nachr. D. König. Gesellsch. D. Wiss. Zu Göttingen, Math-phys. Klasse 1918*, pp. 235–257. An English translation can be found at http://arxiv.org/PS\_cache/physics/pdf/0503/0503066v1.pdf.

<sup>&</sup>lt;sup>2</sup> Dr. Lee Smolin was born in New York City in 1955. He held faculty positions at Yale, Syracuse, and Penn State Universities, where he helped to found the Center for Gravitational Physics and Geometry. In September 2001, he moved to Canada to be a founding member of the Perimeter Institute for Theoretical Physics.

# AN EXAMPLE OF MATHEMATICAL SYMMETRY

Here is a story about Carl Friedrich Gauss (1777–1855). When he was a young child, his teacher wanted to occupy him for a while, so he asked him to add up all the numbers from 1 to 100. That is, find X = 1 + 2 + 3 + ... + 100. To the teacher's surprise, Gauss returned a few minutes later and said that the sum was 5050.

Apparently Gauss noticed that the sum is the same regardless of whether the terms are added forward (from first to last) or backward (from last to first). In other words, X = 1 + 2 + 3 + ... + 100 = 100 + 99 + 98 + ... + 1. If we then add these two ways together, we get

 $X = 1 + 2 + 3 + \dots + 100$   $X = 100 + 99 + 98 + \dots + 1$  $2X = 101 + 101 + \dots + 101$ 

So  $2X = 100 \times 101$  and  $X = (100 \times 101)/2 = 5050$ . Gauss had found a mathematical symmetry, and it tremendously simplified the problem. What is conserved here? It is the sum, *X*. It does not change no matter how you add the numbers.

Wable 4.4 Deletion of Concentration Lower to Mathematical Groups star			
Table 4.1 Relation of Conservation Laws to Mathematical Symmetry			
Conservation Law	Mathematical Symmetry		
Linear momentum	The laws of physics are the same regardless of where we are in space. This positional symmetry implies that linear momentum is conserved.		
Angular momentum	The laws of physics are the same if we rotate about an axis. This rotational symmetry implies that angular momentum is conserved.		
Energy	The laws of physics do not depend on what time it is. This temporal symmetry implies the conservation of energy.		
Electric charge	The interactions of charged particles with an electromagnetic field remain the same if we multiply the fields by a complex number e <sup>iq</sup> . This implies the conservation of charge.		

In summary, Emmy Noether's theorem shows us that (Table 4.1)

- Symmetry under translation produces the *conservation of linear momentum*.
- Symmetry under rotation produces the *conservation of angular momentum*.
- Symmetry in time produces the *conservation of energy*.
- Symmetry in magnetic fields produces the conservation of charge.

# **4.3 THE FIRST LAW OF THERMODYNAMICS**

In this chapter, we focus our attention on the detailed structure of the first law of thermodynamics. To completely understand this law, we need to study a variety of work and heat energy transport modes and to investigate the basic elements of energy conversion efficiency. An effective general technique for solving thermodynamics problems is presented and illustrated. This technique is used in Chapters 5 and 6 and the remainder of the book.

The simplest, most direct statement of the first law of thermodynamics is that *energy is conserved*. That is, energy can be neither created nor destroyed. The condition of zero energy production was expressed mathematically in Eq. (2.15):

$$E_P = 0 \tag{2.15}$$

By differentiating this with respect to time, we obtain an equation for the condition of a zero energy production *rate*:

$$\frac{dE_P}{dt} = \dot{E}_p = 0 \tag{2.16}$$

Whereas Eqs. (2.15) and (2.16) are accurate and concise statements of the first law of thermodynamics, they are relatively useless by themselves, because they do not contain terms that can be used to calculate other variables. However, if these equations are substituted into the energy balance and energy rate balance equations, then the following equations result. For the energy balance,

 $E_G = E_T + E_P$  (as required by the first law)

or

$$E_{\rm G} = E_{\rm T} \tag{4.1}$$

The energy rate balance is

 $\dot{E}_G = \dot{E}_T + \dot{E}_P$  (as required by the first law)

or

and

$$\dot{E}_G = \dot{E}_T \tag{4.2}$$

From now on, we frequently use the phrases *energy balance* and *energy rate balance* in identifying the proper equation to use in an analysis. So, for simplicity, we introduce the following abbreviations:

#### EB = energy balance

#### ERB = energy rate balance

In Chapter 3, we introduce the components of the total system energy *E* as the internal energy *U*, the kinetic energy  $mV^2/2g_{cr}$  and the potential energy  $mgZ/g_{cr}$  or<sup>3</sup>

$$E = U + \frac{mV^2}{2g_c} + \frac{mgZ}{g_c}$$
(3.9)

In this equation, *V* is the magnitude of the velocity of the center of mass of the entire system, *Z* is the height of the center of mass above a ground (or zero) potential datum, and  $g_c$  is the dimensional proportionality factor (see Table 1.2 of Chapter 1). In Chapter 3, we also introduce the abbreviated form of this equation:

$$E = U + KE + PE \tag{3.10}$$

and similarly for the specific energy e,

$$e = \frac{E}{m} = u + \frac{V^2}{2g_c} + \frac{gZ}{g_c}$$
(3.12)

and

$$e = u + \mathrm{ke} + \mathrm{pe} \tag{3.13}$$

In these equations, we continue the practice introduced in Chapter 2 of using uppercase letters to denote *extensive* properties and lowercase letters to denote *intensive* (specific) properties. The energy concepts described in these equations are illustrated in Figure 4.1.

In equilibrium thermodynamics, the proper energy balance is given by Eq. (4.1), where the gain in energy  $E_G$  is to be interpreted as follows. The system is initially in some equilibrium state (call it state 1), and after the application of some "process," the system ends up in a different equilibrium state (call it state 2). If we now add a subscript to each symbol to denote the state at which the property is to be evaluated ( $E_1$  is the total energy of the system in state 1 and so forth), then we can write the energy gain of the system as

$$E_G$$
 = Final total energy – Initial total energy (4.3)

or

or

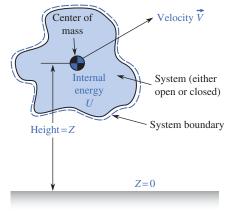
$$E_{C} = E_{2} - E_{1} \tag{4.4}$$

and extending this to Eq. (3.9), we obtain

<sup>3</sup> In this text, we use the symbol V to represent the magnitude of the average velocity |V|, and the symbol  $\forall$  to represent volume.

$$E_{G} = U_{2} - U_{1} + \frac{m}{2g_{c}}(V_{2}^{2} - V_{1}^{2}) + \left(\frac{mg}{g_{c}}\right)(Z_{2} - Z_{1})$$
(4.5)

$$E_{\rm G} = m \left[ u_2 - u_1 + \frac{V_2^2 - V_1^2}{2g_c} + \frac{g}{g_c} (Z_2 - Z_1) \right]$$
(4.6)



**FIGURE 4.1** 

System energy components.



alternatively,

$$E_G = U_2 - U_1 + KE_2 - KE_1 + PE_2 - PE_1$$
(4.7)

and

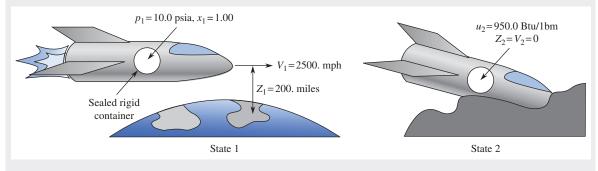
$$E_G = m(u_2 - u_1 + ke_2 - ke_1 + pe_2 - pe_1)$$
(4.8)

In most of the engineering situations we encounter, either the system is not moving at all or it is moving without any change in velocity or height. In these cases,

$$E_G = U_2 - U_1 = m(u_2 - u_1) = E_T$$

# EXAMPLE 4.1

Figure 4.2 shows that 3.00 lbm of saturated water vapor at 10.0 psia is sealed in a rigid container aboard a spaceship traveling at 25,000. mph at an altitude of 200. mi. What energy transport is required to decelerate the water to zero velocity and bring it down to the surface of the Earth such that its final specific internal energy is 950.0 Btu/lbm? Neglect any change in the acceleration of gravity over this distance.



## FIGURE 4.2

Example 4.1.

### Solution

Let the system in this example be just the water in the container, then the process followed by the water is a constant volume process (the water is in a "rigid, sealed container"). Therefore, the problem statement can be outlined as follows:

$$\begin{array}{ccc} \underline{\text{State 1}} & \underline{m = 3.0 \text{ lbm}, \forall = \text{ constant}} \\ p_1 = 10.0 \text{ psia} & \underline{x_1 = 1.00(\text{saturated vapor})} \\ v_1 = v_e(\text{at } 10.0 \text{ psia}) = 38.42 \text{ ft}^3/\text{lbm} \end{array} \xrightarrow{\begin{array}{c} \underline{\text{M} = 3.0 \text{ lbm}, \forall = \text{ constant}} \\ \underline{w_2 = 050.0 \text{ Btu/lbm}} \\ \underline{v_2 = v_1 = 38.42 \text{ ft}^3/\text{lbm}} \end{array}$$

Notice how the process path gives us the value of a property ( $\nu_2$ ) in the final state. To determine the required energy transport, we use the energy balance Eq. (4.1), along with the definition of the energy gain term  $E_G$  from Eq. (4.5):

EB:  $E_G = E_T + E_{P_0}$  (as required by the first law)

and, assuming g is constant during this process,

$$E_G = E_T = U_2 - U_1 + \frac{m}{2g_c} \left( V_2^2 - V_1^2 \right) + \frac{mg}{g_c} \left( Z_2 - Z_1 \right)$$

Here,  $V_2 = Z_2 = 0$ , so

$$E_T = U_2 - U_1 - \frac{m}{2g_c}V_1^2 - \frac{mg}{g_c}Z_1$$

Table C.2a in Thermodynamic Tables to accompany Modern Engineering Thermodynamics gives

$$u_1 = u_g(10.0 \text{ psia}) = 1072.2 \text{ Btu/lbm}$$

and the problem statement requires that  $u_2 = 950.0$  Btu/lbm. Therefore,

 $U_1 = mu_1 = (3.00 \text{ lbm})(1072.2 \text{ Btu/lbm}) = 3216.6 \text{ Btu}$ 

(Continued)

# EXAMPLE 4.1 (Continued)

and

$$U_2 = mu_2 = (3.00 \text{ lbm})(950.0 \text{ Btu/lbm}) = 2850 \text{ Btu}$$

so

$$E_{\rm T} = (2850 - 3216.6) \text{ Btu} - \frac{3.00 \text{ lbm}}{2} \left[ (25,000. \text{ mile/h}) \left( \frac{5280 \text{ ft/mile}}{3600 \text{ s/h}} \right) \right]^2$$

$$\times \frac{\frac{1 \text{ Btu}}{778.16 \text{ ft} \cdot \text{lbf}}}{32.174 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}} - \frac{3.00 \text{ lbm}(32.174 \text{ ft/s}^2)}{(32.174 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2})} (200. \text{ miles})(5280 \text{ ft/mile}) \left( \frac{1 \text{ Btu}}{778.16 \text{ ft} \cdot \text{lbf}} \right)$$

$$= -366.6 - 80,550 - 4071 = -85,000 \text{ Btu} \text{ (to three significant figures)}$$

Therefore, 85,000 Btu of energy must be transferred *out of* the water ( $E_T$  is negative here) by some mechanism. This can be done, for example, by having the spaceship (and the water) do work on the atmosphere by aerodynamic drag as it lands.

#### Exercises

- **1.** What would be the value of  $u_2$  in Example 4.1 if  $E_T$  were zero? Answer:  $u_2 = 29,300$  Btu/lbm. (What is the physical state of the water now?)
- **2.** Which causes the larger change in  $E_G$ :
  - **a.** A velocity increase from 0 to 1 ft/s or an increase in height from 0 to 1 ft?
  - **b.** A velocity increase from 0 to 100 ft/s or a height increase from 0 to 100 ft?
  - Answers: (a) height, (b) velocity.
- 3. Determine the value of  $E_T$  that must occur when you stop a 1300. kg automobile traveling at 100. km/h on a level road with no change in internal energy. Answer:  $E_T = 502$  kJ.

In nonequilibrium systems, we use the energy rate balance equation with  $\dot{E}_G$  defined as

$$\dot{E}_G = \frac{d}{dt} \left( U + \frac{m}{2g_c} V^2 + \frac{mg}{g_c} Z \right)_{\text{system}} = \dot{E}_T$$
(4.9)

Equation (4.9) can become quite complicated for open systems whose total mass is rapidly changing (such as with rockets), because it expands as follows (using U = mu):

$$\dot{E}_{G} = m \left[ \dot{u} + \frac{V}{g_{c}} (\dot{V}) + \frac{g}{g_{c}} (\dot{Z}) \right] + \left( u + \frac{V^{2}}{2g_{c}} + \frac{gZ}{g_{c}} \right) \dot{m} = \dot{E}_{T}$$
(4.10)

Notice that, in this equation,  $\dot{V} = dV/dt$  is the magnitude of the instantaneous acceleration, and  $\dot{Z}$  is the magnitude of the instantaneous vertical velocity.

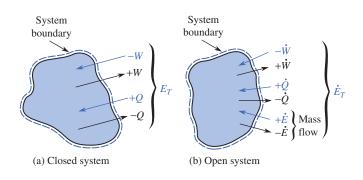
The equilibrium thermodynamics energy balance and the nonequilibrium energy rate balance are fairly simple concepts; however, their implementation can be quite complex. Each of the gain, transport, and production terms may expand into many separate terms, all of which must be evaluated in an analysis. Next, we investigate the structure of the energy transport and energy transport rate terms.

# 4.4 ENERGY TRANSPORT MECHANISMS

There are three energy transport mechanisms, any or all of which may be operating in any given system: (1) heat, (2) work,<sup>4</sup> and (3) mass flow. These three mechanisms and their sign conventions are illustrated in Figure 4.3.

Note that the sign conventions for heat and work shown in Figure 4.3 are not the same. Heat transfer *into* a system is taken as positive, whereas work must be produced by or come *out of* a system to be positive. This is the conventional mechanical engineering sign convention and reflects the traditional view that heat coming out

<sup>&</sup>lt;sup>4</sup> The types of work transports of energy included here are only those due to dissipative or nonconservative forces. For example, the work associated with gravitational or electrostatic forces is not considered a work mode because it is conservative (i.e., it is representable by the gradient of a scalar quantity) and is consequently nondissipative. Energy transports resulting from the actions of conservative forces have their own individual terms in the energy balance equation (such as  $mgZ/g_c$  for the gravitational potential energy).



#### FIGURE 4.3

Energy transport mechanisms.

# WHAT ARE HEAT AND WORK ANYWAY?

*Heat* is usually defined as energy transport to or from a system due to a temperature difference between the system and its surroundings. This can occur by only three modes: conduction, convection, and radiation.

*Work* is more difficult to define. It is often defined as a force moving through a distance, but this is only one type of work; there are many other work modes as well. Since the only energy transport modes for moving energy across a system's boundary are heat, mass flow, and work, the simplest definition of work is that it is any energy transport mode that is neither heat nor mass flow.<sup>5</sup>

<sup>5</sup> Work can also be defined using the concept of a "generalized" force moving through a "generalized" displacement, see Table 4.2 later in this chapter.

of a system is "lost" (i.e., negative), while work produced by a system (such as an engine) should be assigned a positive value.

By definition, a closed system has no mass crossing its system boundary, so it can experience only work and heat transport mechanisms. Also, since the gain, transport, and production terms in the balance equation are defined to be *net* values (see Eq. (2.10)), we define

- **1.** The *net* heat transport of energy *into* a system =  $\sum_i Q_i = Q$  and the *net* heat transport rate of energy *into* a system =  $\sum_i \dot{Q}_i = \dot{Q}$ .
- **2.** The *net* work transport of energy *out* of a system =  $\sum_i W_i = W$  and the *net* work transport rate of energy *out* of a system =  $\sum_i \dot{W}_i = \dot{W}$ .
- **3.** The *net* mass transport of energy *into* the system =  $\sum_i E_i = \sum E_{\text{mass flow}}$  and the *net* mass transport rate of energy *into* the system =  $\sum_i \dot{E}_i = \sum \dot{E}_{\text{mass flow}}$ .

Thus, for a *closed system*, the total energy transport becomes

$$E_T = Q - W \tag{4.11}$$

and the total energy transport rate is

$$\dot{E}_T = \dot{Q} - \dot{W} \tag{4.12}$$

For open systems, the same quantities are

$$E_T = Q - W + \sum_{\text{flow}} E_{\text{flow}}$$
(4.13)

and

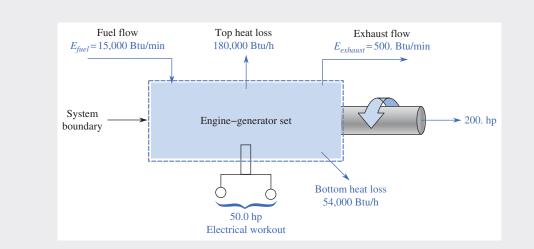
$$\dot{E}_T = \dot{Q} - \dot{W} + \sum \dot{E}_{\text{flow}}$$
(4.14)

In Eqs. (4.13) and (4.14), note that we write the summation signs on the net mass transport of energy terms, but for simplicity, we do not write the summation signs on the work or heat transport terms. This is because you often have open systems with more than one mass flow stream, but seldom do you have more than one

type of work or heat transport present. However, you must always remember that W,  $\dot{W}$ , Q, and  $\dot{Q}$  are also *net* terms and represent a summation of all the different types of work and heat transports of energy present. This is illustrated in the following example.

# **EXAMPLE 4.2**

Determine the energy transport rate for the system shown in Figure 4.4.



## FIGURE 4.4

Example 4.2

#### Solution

From Eq. (4.14), the total energy transport rate is

$$\dot{E}_{\rm T} = \dot{Q} - \dot{W} + \sum \dot{E}_{\rm flow}$$

where

 $\dot{Q}$  = net heat transfer *into* the system = -180.×10<sup>3</sup> Btu/h - 54.0×10<sup>3</sup> Btu/h = -234×10<sup>3</sup> Btu/h

and

 $\dot{W}$  = net work rate *out of* the system = 200. hp + 50.0 hp = 250. hp

while

$$\sum E_{\text{flow}}$$
 = net mass flow of energy *into* the system  
= 15.0 × 10<sup>3</sup> Btu/min - 500.Btu/min = 14.5 × 10<sup>3</sup> Btu/min

So

$$E_{\rm T} = (-234 \times 103 \text{ Btu/h})[1 \text{ h}/(60 \text{ min})] - (250 \text{ hp})[42.4 \text{ Btu}/(\text{hp} \cdot \text{min})] + 14.5 \times 103 \text{ Btu/min} = 0.00 \text{ Btu/min}$$

#### Exercises

- **4.** Determine the energy transport rate that occurs in Example 4.2 when the work mode directions are reversed. **Answer**:  $\dot{E}_{\rm T} = 21.2 \times 10^3$  Btu/min.
- 5. Determine the net rate of energy gain of a closed system that receives heat at a rate of 4500. kJ/s and produces work at a rate of 1500. kJ/s. Answer:  $\dot{E}_{G} = 3000$ . KJ/s.
- 6. An insulated open system has a net gain of 700. Btu of energy while producing 500. Btu of work. Determine the mass flow energy transport. Answer:  $E_{mass flow} = 1.20 \times 10^3$  Btu.

The system of Example 4.2 has no *net* energy transport rate, even though it has six energy transport rates. Note that the energy rate balance (Eq. (4.2)) for this system is  $\dot{E}_G = \dot{E}_T$ ; therefore, this system also has no net gain of energy. That is, the total energy *E* of this system is constant in time.

# **4.5 POINT AND PATH FUNCTIONS**

A quantity, say y, that has a value at every point within its range is called a *point function*. Its derivative is written as dy, and its integral from state 1 to state 2 is

$$\int_1^2 dy = y_2 - y_1$$

Thus, the value of the integral depends only on the values of *y* at the end points of the integration path and is independent of the actual path taken between these end points. This is a fundamental characteristic of point functions. *All intensive and extensive thermodynamic properties are point functions*. Therefore, we can write

$$\int_{1}^{2} dE = E_{2} - E_{1}; \quad \int_{1}^{2} du = u_{2} - u_{1}; \quad \int_{1}^{2} dm = m_{2} - m_{1}$$

and so forth.

A quantity, say *x*, whose value depends on the *path* taken between two points within its range is called a *path function*. Since path functions do not differentiate or integrate in the same manner as point functions, we cannot use the same differential and integral notation for both path and point functions. Instead, we let  $\overline{dx}$  denote the differential of the path function *x*, and we define its integral over the path from state 1 to state 2 as

$$\int_{1}^{2} \overline{dx} = {}_{1}x_{2} \quad \left[ \text{Note:} \int_{1}^{2} \overline{dx} \neq (x_{2} - x_{1}) \right]$$
(4.15)

A path function does not have a value at a point. It has a value only for a path of points, and this value is directly determined by all the points on the path, not just its end points. For example, the area A under the curve of the point function w = f(y) is a path function because

$$\overline{d}A = w \, dy = f(y) \, dy$$

and

$$\int_{1}^{2} \overline{d}A = {}_{1}A_{2} = \int_{Y_{1}}^{Y_{2}} f(y) \, dy = \text{area under} f(y) \text{ between the points } y_{1} \text{ and } y_{2}$$

Clearly, if the path f(y) is changed, then the area  $_1A_2$  is also changed. Consequently, we say that  $_1A_2$  is a path function.

We see in the next sections that both the work and heat transports of energy are path functions. Therefore, we write the differentials of these quantities as  $\overline{d}W$  and  $\overline{d}Q$ , and their integrals as

$$\int_{1}^{2} \overline{d}W = {}_{1}W_{2} \tag{4.16}$$

and

NOTE!

$$\int_{1}^{2} \overline{d}Q = {}_{1}Q_{2} \tag{4.17}$$

Since the associated rate equations contain the time differential, we define *power* as the work rate, or

$$\dot{W} = \overline{d}W/dt \tag{4.18}$$

and, similarly, the heat transfer rate is

$$\dot{Q} = \overline{d}Q/dt \tag{4.19}$$

Each of the different types of work or heat transport of energy is called a *mode*. A system that has no operating work modes is said to be aergonic. Similarly, a system that changes its state without any work transport of energy having

Since work and heat are not thermodynamic properties and therefore not point functions,  $\int_{1}^{2} \overline{d}W \neq W_{2} - W_{1}$  and  $\int_{1}^{2} \overline{d}W \neq \Delta W$ . Similarly,  $\int_{1}^{2} \overline{d}Q \neq Q_{2} - Q_{1}$ , and  $\int_{1}^{2} \overline{d}Q \neq \Delta Q$ . Equations (4.16) and (4.17) are the only correct ways to write these path function integrals.

# WHAT IS AERGONIC ANYWAY?

The term aergonic comes from the Greek roots *a* meaning "not" and *ergon* meaning "work," and it should be interpreted to mean "no work has occurred." It is the analog of the word *adiabatic*, meaning no heat transfer has occurred, introduced later in this chapter.

Substituting Eqs. (4.8) and (4.11) into Eq. (4.1) and rearranging gives the general closed system energy balance equation for a system undergoing a process from state 1 to state 2 as

General closed system energy balance:  

$${}_{1}Q_{2} - {}_{1}W_{2} = (E_{2} - E_{1})_{\text{system}}$$

$$= m[(u_{2} - u_{1}) + (V_{2}^{2} - V_{1}^{2})/(2g_{c}) + (Z_{2} - Z_{1})g/g_{c}]_{\text{system}}$$
(4.20)

and substituting Eq. (4.10) with m = constant and Eq. (4.12) into Eq. (4.2) gives the general closed system energy rate balance as

General closed system energy *rate* balance:  $\dot{Q} - \dot{W} = (dE/dt)_{\text{system}} = (m\dot{u} + mV\dot{V}/g_c + mg \dot{Z}/g_c)_{\text{system}}$ (4.21)

Similarly, substituting Eqs. (4.9) and (4.14) into Eq. (4.2) gives the general open system energy rate balance as

General open system energy rate balance:

$$-\dot{W} + \sum \dot{E}_{\text{flow}} = (d/dt)(mu + mV^2/2g_c + mZg/g_c)_{\text{system}}$$

$$(4.22)$$

where the mass of the system is no longer required to be constant.

occurred is said to have undergone an *aergonic process*. While there are only three modes of heat transport, there are many modes of work transport. In the following segments, four mechanical work modes and five nonmechanical work modes are studied in detail.

## 4.6 MECHANICAL WORK MODES OF ENERGY TRANSPORT

In mechanics, we recognize that work is done whenever a force moves through a distance. When this force is a mechanical force  $\vec{F}$ , we call this work mode *mechanical work* and define it as

$$\left(\overline{d}W\right)_{\text{mechanical}} = \left(\overrightarrow{F}_{\text{applied by the system}}\right) \cdot d\overrightarrow{\mathbf{x}} - \left(\overrightarrow{F}_{\text{applied on the system}}\right) \cdot d\overrightarrow{\mathbf{x}}$$
(4.23)

## **CAN YOU ANSWER THIS QUESTION FROM 1936?**

On page 66 of the October 1936 issue of *Modern Mechanix* is a discussion of the oddities of science that reads: "Modern science states that energy cannot be destroyed. Scientists are now wondering what happens to the energy contained in a compressed spring destroyed in acid." How would you answer this question more than 70 years later?

The person who wrote this in 1936 did not understand the concept of internal energy. Then, neglecting any changes in kinetic or potential energy, an energy balance on the system gives

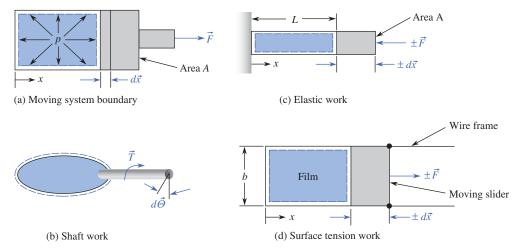
$$_{1}Q_{2} - _{1}W_{2} = (E_{2} - E_{1})_{\text{system}} = (U_{2} - U_{1})_{\text{system}}$$

where  $U_1 = U_{acid} + U_{spring} = (m_{acid}u_{acid} + m_{spring}u_{spring})$ . Now,  $U_{spring} = F(\Delta X)$ , the work done in compressing the spring. Finally,  $U_2 = U_{acid+spring} = (m_{acid} + m_{spring})u_{acid+spring}$ . If we make the reasonable assumption that the spring dissolved without any heat transfer ( $_1Q_2 = 0$ ) and aergonically ( $_1W_2 = 0$ ), then the energy balance equation gives  $U_2 = U_1$ , and solving it for the final specific internal energy of the acid-spring solution, we find that

$$u_{\text{acid}+\text{spring}} = \frac{m_{\text{acid}}u_{\text{acid}} + m_{\text{spring}}u_{\text{spring}}}{m_{\text{acid}} + m_{\text{spring}}}$$

So the answer to the 1936 question is this: *The energy contained in the compressed spring ends up as part of the energy of the combined acid-spring solution*. That is, since the mechanical work that went into compressing the spring ended up as part of the spring's internal energy, when the spring was dissolved in the acid, the internal energy in the spring became part of the internal energy of the acid-spring solution.

Also, if we assume that the acid-spring solution is a simple incompressible liquid with an internal energy that depends only on temperature, then we can write  $u_{acid+spring} = cT$ , and we see that the energy contained in the compressed spring reappears as an increase in the temperature of the resulting acid-spring solution.



## FIGURE 4.5

Four classical types of mechanical work.

or

$$({}_{1}W_{2})_{\text{mechanical}} = \int_{x1}^{x2} (\overrightarrow{F}_{\text{applied by the system}}) \cdot d\overrightarrow{\mathbf{x}} - \int_{x1}^{x2} (\overrightarrow{F}_{\text{applied on the system}}) \cdot d\overrightarrow{\mathbf{x}}$$
(4.24)

Note that our sign convention requires that work done by the system be *positive*, while work done *on* the system be *negative*.

In thermodynamics, the four classical types of mechanical work (Figure 4.5) are

- 1. Moving system boundary work.
- 2. Rotating shaft work.
- 3. Elastic work.
- **4.** Surface tension work.

These are very important work modes in engineering analysis and the following material provides a detailed discussion of their major characteristics.

## 4.6.1 Moving System Boundary Work

Whenever a system boundary moves such that the total volume of the system changes, moving system boundary work occurs. This is sometimes called *expansion* or *compression work*, and it has wide application in mechanical power technology. In this case, the force is applied by the system through the pressure p (see Figure 4.5a), so  $\vec{F} = p\vec{A}$  and  $\vec{F} \cdot d\vec{x} = p\vec{A} \cdot d\vec{x} = pdV$ , where p is the pressure acting on the system boundary,  $\vec{A}$  is the area vector (defined to be normal to the system boundary and pointing outward),  $d\vec{x}$  is the differential boundary movement, and  $d\vec{V}$  is the differential volume  $\vec{A} \cdot d\vec{x}$ . Consequently,

$$(\bar{d}W)_{\text{moving}} = p \, dV \tag{4.25}$$

and for moving boundary work,

Moving boundary work:  

$$\binom{1}{W_2}_{\text{moving boundary}} = \int_1^2 p \, d\Psi$$
(4.26)

# **EXAMPLE 4.3**

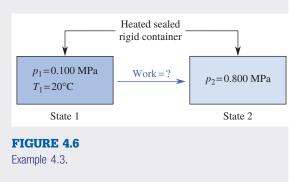
The sealed, rigid tank shown in Figure 4.6 contains air at 0.100 MPa and 20.0 °C. The tank is then heated until the pressure in the tank reaches 0.800 MPa. Determine the mechanical moving boundary work produced in this process.

# **EXAMPLE 4.3** (Continued)

#### Solution

Let the system be the material inside the tank. The process of heating the tank is one of constant volume (the tank is "rigid"). Therefore, since the system volume,  $\forall$ , is constant,  $d \forall = 0$  and the moving boundary work is:

$$\binom{1}{1}W_2_{\text{boundary}} = \int_1^2 p d\Psi = 0$$



Therefore, no moving boundary work occurs during this process.

Since a "rigid" container cannot change its volume, its moving boundary work is always zero regardless of the process it undergoes.

# **EXAMPLE 4.4**

The weather balloon in Figure 4.7 is inflated from a constant pressure, compressed gas source at 20.0 psia. Determine the moving system boundary work as the balloon expands from a diameter of 1.00 ft to 10.0 ft.

#### Solution

Assume the balloon is a sphere, then  $\Psi = 4/_3 \pi R^3 = 1/_6 \pi D^3$ . The process here is one of constant pressure, so *p* = constant, and

 $p_1=20.0 \text{ psia}$   $D_1=1.00 \text{ ft dia}$   $D_2=10.0 \text{ ft dia}$   $D_2=10.0 \text{ ft dia}$  $D_2=10.0 \text{ ft dia}$ 

 $p_2 = 20.0 \text{ psia}$ 

The work is positive because the balloon does work on the atmosphere as it expands and pushes the atmosphere out of the way.



#### Exercises

- 7. In Example 4.3, is the moving boundary work always zero for a sealed, rigid container? Are any other work modes always zero for this type of system? Could a piston-cylinder apparatus be modeled as a sealed, rigid system? Answers: Yes, no, no. (It is sealed and the components, the piston and the cylinder, are rigid, but the piston can move, producing a change in the enclosed volume.)
- 8. Determine the moving boundary work for the balloon in Example 4.4 as it deflates from a diameter of 10. ft to a diameter of 5.0 ft at a constant pressure of 20. psia. What does the work on the balloon? Answer:  $({}_{1}W_{2})_{moving boundary} = -1.3 \times 10^{6}$  ft · lbf. The surrounding atmosphere does work on the balloon as it deflates, that is why the work is negative.
- **9.** If the pressure inside a system depends on volume according to the relation  $p = K_1 + K_2 \Psi + K_3/\Psi$ , where  $K_1$ ,  $K_2$ , and  $K_3$  are constants, determine the appropriate equation for the moving boundary work done as the volume changes from  $\Psi_1$  to  $\Psi_2$ . **Answer**:  $({}_1W_2)_{\text{moving boundary}} = K_1(\Psi_2 \Psi_1) + K_2(\Psi_2^2 \Psi_1^2)/2 + K_3 \ln(\Psi_2/\Psi_1)$ .

To carry out the integration indicated in Eq. (4.26), the exact p = p(V) pressure volume function must be known. This function is usually given in the process path specification of a problem statement. For example, in Example 4.3, the process is one of constant volume (the container is rigid), so dV = 0; and in Example 4.4, the filling process is isobaric (p = constant), so the integral of Eq. (4.26) is very easy. In general, outside of these two cases, the integration of Eq. (4.26) is not trivial and must be determined with great care.

As an example of a nontrivial integration of Eq. (4.26), consider a process that obeys the relation

$$p \Psi^n = \text{constant}$$
 (4.27)

or

$$p_1 \underbrace{\Psi_1^n}_1 = p_2 \underbrace{\Psi_2^n}_2$$

where the exponent n is a constant. Such processes are called *polytropic processes*.<sup>6</sup> The moving system boundary work of any substance undergoing a polytropic process is

$$({}_{1}W_{2})_{\text{polytropic}}_{\text{moving boundary}} = \int_{1}^{2} p d\Psi = \int_{1}^{2} \frac{\text{constant}}{\Psi^{n}} d\Psi$$

For n = 1, this integral becomes

$$\binom{W_2}{polytropic (n=1)}_{\text{moving boundary}} = p_1 \Psi_1 \ln \frac{\Psi_2}{\Psi_1} = p_2 \Psi_2 \ln \frac{\Psi_2}{\Psi_1}$$
(4.28)

and for  $n \neq 1$ , it becomes

$$\binom{W_2}{polytropic (n\neq 1),}_{moving boundary} = \frac{p_2 \Psi_2 - p_1 \Psi_1}{1 - n}$$
(4.29)

If the material undergoing a polytropic process is an ideal gas, then it must simultaneously satisfy both of the following equations:

- **1.** The ideal gas equation of state,  $p\Psi = mRT$ .
- **2.** The polytropic process equation,  $p \Psi^n = \text{constant}$ .

Combining these two equations by eliminating the pressure p gives

$$mRT \Psi^{n-1} = \text{constant}$$

or, for a fixed mass system,

$$T_1 \mathcal{V}_1^{n-1} = T_2 \mathcal{V}_2^{n-1}$$

or

$$\frac{T_2}{T_1} = \left(\frac{\Psi_2}{\Psi_1}\right)^{1-n} = \left(\frac{\nu_2}{\nu_1}\right)^{1-n}$$
(4.30)

Similarly, eliminating  $\forall$  in these two equations (for a fixed mass system) gives the polytropic process equations for an ideal gas:

## Polytropic process equations for an ideal gas

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = \left(\frac{\nu_2}{\nu_1}\right)^{1-n}$$
(4.31)

Finally, if we have an ideal gas undergoing a polytropic process with  $n \neq 1$ , then its moving system boundary work is given by Eq. (4.29), with  $p_2 \Psi_2 - p_1 \Psi_1 = mR(T_2 - T_1)$  as the polytropic work equation for an ideal gas  $(n \neq 1)$ :

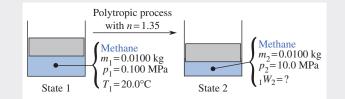
## Polytropic work equation for an ideal gas $(n \neq 1)$

$$\binom{W_2}{1}_{\substack{\text{polytropic} (n \neq 1) \\ \text{ideal gas} \\ \text{moving boundary}}} = \frac{mR}{1-n} (T_2 - T_1)$$
(4.32)

<sup>&</sup>lt;sup>6</sup> The term *polytropic* comes from the Greek roots *poly* meaning "many" and *trope* meaning "turns" or "paths."

# **EXAMPLE 4.5**

Figure 4.8 shows a new process in which 0.0100 kg of methane (an ideal gas) is compressed from a pressure of 0.100 MPa and a temperature of 20.0 °C to a pressure of 10.0 MPa in a polytropic process with n = 1.35. Determine the moving boundary work required.



#### **FIGURE 4.8**

Example 4.5.

#### Solution

Since the methane behaves as an ideal gas and  $n \neq 1$ , we can find the work required from Eq. (4.32):

$$({}_{1}W_{2})_{\substack{\text{polytropic}(n\neq 1)\\\text{ideal gas}}} = \frac{mR}{1-n}(T_{2}-T_{1})$$

where the value of  $T_2$  can be found from Eq. (4.31):

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = (20.0 + 273.15 \text{ K}) \left(\frac{10.0 \text{ MPa}}{0.100 \text{ MPa}}\right)^{(1.35-1)/1.35} = 967 \text{ K} = 694 \text{ °C}$$

Using Table C.13b of *Thermodynamic Tables to accompany Modern Engineering Thermodynamics* to find the value of the gas constant for methane,  $R_{\text{methane}} = 0.518 \text{ kJ/kg} \cdot \text{K}$ , Eq. (4.32) then gives

$$\binom{1}{1} W_2 \underset{\text{moving boundary}}{\text{polytropic } (n \neq 1)} = \frac{(0.0100 \text{ kg})(0.518 \text{ kJ/kg} \cdot \text{K})}{1 - 1.35} (967 - 293.15) = -9.98 \text{ kg}$$

The work comes out negative, because it is being done on the system.

#### Exercises

10. Determine the work required in Example 4.5 if the final pressure of the methane is 0.500 MPa. Answer: -2.25 kJ.

- **11.** If the work required in Example 4.5 is -5.00 kJ, determine the final temperature and pressure of the methane. Answer:  $T_2 = 631 \text{ K}$ ,  $p_2 = 1.92 \text{ MPa}$ .
- **12.** If the gas used in Example 4.5 were air, determine the work required to compress it polytropically from 14.7 psia, 70.0°F to 150.°F with n = 1.33. Answer:  $_1W_2 = -285.1$  ft lbf

## 4.6.2 Rotating Shaft Work

Whenever a rotating shaft carrying a torque load crosses a system boundary, *rotating shaft work* is done. In this case (see Figure 4.5b),

$$(\vec{d}W)_{\text{rotating}} = \vec{T} \cdot d\vec{\theta}$$
(4.33)

and, for rotating shaft work,

Rotating shaft work  

$$\binom{W_2}{({}_{1}W_2)_{\text{rotating shaft}}} = \int_{1}^{2} \overrightarrow{T} \cdot d\overrightarrow{\theta}$$
(4.34)

where  $\vec{T}$  is the torque vector produced by the system on the shaft and  $d\vec{\theta}$  is its angular displacement vector. These two vectors are in the direction of the shaft axis. Normally, thermodynamic problem statements do not require rotating shaft work to be calculated from Eq. (4.34). The rotating shaft work is usually openly given as part of the problem statement. For example, if you are analyzing an automobile internal combustion engine producing 150. ft·lbf of work at the crankshaft, you must be able to recognize that  $(_1W_2)_{\text{rotating shaft}} = 150$ . ft·lbf.

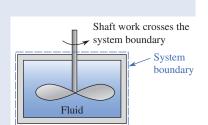
# WHEN IS SHAFT WORK NOT SHAFT WORK?

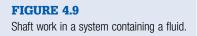
Suppose you have a system that contains a fluid, and this fluid is in contact with a mixing blade or an impeller driven by a shaft passing through the system boundary (see Figure 4.9). This would constitute an example of shaft work.

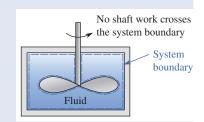
The shaft and the blade or impeller are inside the system and their physical and thermodynamic properties are part of the system's properties. You have a heterogenous system made up of the fluid and the solid shaft and blade. If the mass of the fluid is large enough and the size of the shaft and blade is small enough, then their impact on the system's properties can be neglected and the system can be considered to consist of the fluid alone. However, this is not always the case. Suppose now you exclude the shaft and the blade or impeller from the system by restricting the system to be only the fluid and redraw the system boundaries so that they pass along the surface of the shaft and blade (see Figure 4.10). Now, your system consists of a pure substance (the fluid), but what kind of work mode do you now have?

Since the only work modes we can analyze are "reversible," the fluid medium cannot possess viscosity (fluid friction), and consequently, there can be no shear forces on the blade. The only force a viscousless fluid can exert on the blades is a pressure force, *p*. As the blade moves, the system boundary must move accordingly to keep up with it, and the pressure force on the blade must also move. This is just the definition of the *moving boundary* work mode. Consequently, this type of shaft work is not really shaft work at all, it is really moving boundary work.

Another example is the shaft work from an internal combustion engine. It is produced inside the engine by moving boundary piston-cylinder work, and in a frictionless reversible engine, these two work modes are equivalent. However, in a real engine, where friction and other losses are present, these two work modes are not equivalent (see Figure 4.11).







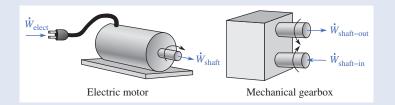


Not all shaft work can be viewed as moving boundary work. The shaft work from an electric motor or a mechanical gearbox is not equivalent to moving boundary work (see Figure 4.12).



#### FIGURE 4.11

Reversible and irreversible work in an IC engine.



## **FIGURE 4.12**

Shaft work from systems without internal moving boundaries.

## **4.6.3 Elastic Work**

Whenever we compress or extend an elastic solid (like a spring), we perform elastic work. Consider a force  $\pm \vec{F}$  applied on the end of an elastic rod (see Figure 4.5c). The normal stress  $\sigma$  in the rod is

$$\sigma = \pm \frac{|\vec{F}|}{A} \tag{4.35}$$

where  $|\vec{F}|$  is the magnitude of the force and A is the cross-sectional area of the rod. Since the force  $\vec{F}$  and its corresponding displacement  $d\vec{x}$  are always in the same direction, the vector dot product  $\vec{F} \cdot d\vec{x}$  always reduces to Fdx, where  $F = |\vec{F}|$  and  $dx = |d\vec{x}|$ , and when the force is applied on the system from the surroundings rather than being produced by the system, the work is negative and its increment is

$$\overline{dW} = -\overline{F} \cdot d\overline{x} = -Fdx = -\sigma Adx \tag{4.36}$$

The strain  $\varepsilon$  in the rod is defined as

$$d\varepsilon = \frac{dx}{L} = \frac{A \, dx}{AL} = \frac{A \, dx}{\Psi} = \frac{d\Psi}{\Psi} \tag{4.37}$$

where *L* is the length of the rod and *AL* is its volume  $\forall$ . Then,

$$Adx = d\Psi = \Psi d\varepsilon \tag{4.38}$$

and Eq. (4.36) becomes

$$dW = -\sigma A dx = -\sigma \Psi d\varepsilon \tag{4.39}$$

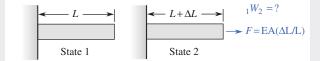
Therefore, for elastic work,

Elastic work  

$$({}_{1}W_{2})_{\text{elastic}} = -\int_{1}^{2} \sigma \forall d\varepsilon$$
(4.40)

## EXAMPLE 4.6

Determine an expression for the work involved in deforming a constant volume elastic solid that obeys Hooke's law of elasticity (see Figure 4.13).



#### FIGURE 4.13

Example 4.6.

## Solution

Here we have V = constant. Also, from strength of materials we can write Hooke's law as  $\sigma = E\varepsilon$ , where *E* is Young's modulus of elasticity. Then, Eq. (4.40) becomes

$$({}_{1}W_{2})_{\text{elastic}} = -\int_{1}^{2} \sigma \Psi \, d\varepsilon = -\int_{1}^{2} E \Psi \varepsilon \, d\varepsilon = -E \Psi \int_{1}^{2} \varepsilon \, d\varepsilon$$
$$= -E \Psi \left(\frac{\varepsilon_{2}^{2} - \varepsilon_{1}^{2}}{2}\right) = -\frac{\Psi}{2E} (\sigma_{2}^{2} - \sigma_{1}^{2})$$

Thus, if  $\varepsilon_2^2 > \varepsilon_1^2$ , then  $(_1W_2)_{\text{elastic}}$  is negative and work is being put into the system; and if  $\varepsilon_2^2 < \varepsilon_1^2$ , then  $(_1W_2)_{\text{elastic}}$  is positive and work is being produced by the system. Note that both tensile strains ( $\varepsilon > 0$ ) and compressive strains ( $\varepsilon < 0$ ) are possible here. But, the resulting work formula deals only with  $\varepsilon^2$  and consequently gives the correct result regardless of the strain direction.

#### Exercises

- **13**. What type of rigid system has zero elastic work regardless of the loading? **Answer**: A perfectly rigid system  $(E = \infty)$ .
- **14.** If the system analyzed in Example 4.6 was a rectangular steel bar, 1.0 inch square by 12 inches long, determine the elastic work required to stress it from 0.0 to  $10 \times 10^3$  lbf/in<sup>2</sup>. Use  $E_{\text{steel}} = 30 \times 10^6$  lbf/in<sup>2</sup>. Answer:  $(_1W_2)_{\text{elastic}} = -1.7$  ft·lbf.
- **15.** Ten joules of elastic work is applied to a circular brass rod 0.0100 m in diameter and 1.00 m long. Determine the resulting stress and strain in the bar if it is initially unloaded. Use  $E_{\text{brass}} = 1.05 \times 10^{11}$  Pa. **Answer**:  $\sigma = 164$  MPa and  $\varepsilon = 1.56 \times 10^{-3}$  m/m.

## 4.6.4 Surface Tension Work

Surface tension work is the two-dimensional analog of the elastic work just considered. Figure 4.5d shows a soap film on a wire loop. One side of the loop has a movable wire slider that can either compress or extend the film. As in the case of the elastic solid, the force and deflection are always in the same direction and the force is applied *to* the system, so we can modify Eq. (4.36) to read

$$\overline{d}W = -\overline{F} \cdot d\overline{x} = -F \, dx = -(2\sigma_{\rm s}b) \, dx \tag{4.41}$$

where  $\sigma_s$  is the surface tension of the film, and *b* is the length of the moving part of the film. The factor of 2 appears because the film normally has two surfaces (top and bottom) in contact with air. Now,  $2b \cdot dx = dA =$  change in the film's surface area, so Eq. (4.41) becomes

$$\overline{d}W = -\sigma_s dA \tag{4.42}$$

and, for the surface tension work,

## Surface tension work

$$({}_{1}W_{2})_{\text{surface}} = -\int_{1}^{2} \sigma_{s} \, dA \tag{4.43}$$

## EXAMPLE 4.7

Determine the amount of surface tension work required to inflate the soap bubble shown in Figure 4.14 from a diameter of zero to 0.0500 m. The surface tension of the soap film can be taken to be a constant 0.0400 N/m.

#### Solution

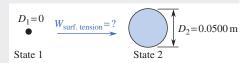
Here,  $\sigma_s = \text{constant} = 0.0400 \text{ N/m}$ . Note that we are not calculating the surface area of the bubble here from its geometric elements, but wish only to find the change in area between states 1 and 2. Consequently, the area integral in this instance can be treated as a point function rather than as a path function. So Eq. (4.43) becomes

$$({}_1W_2)_{\text{surface}\atop{\text{tension}}} = -\sigma_s \int_1^2 dA = -\sigma_s (A_2 - A_1)$$

where  $A_1 = 0$ . Now, since a soap bubble has *two* surfaces (the outside and inside films),

$$A_2 = 2(4\pi R^2) = 2(4\pi) \left(\frac{0.0500 \text{ m}}{2}\right)^2 = 0.0157 \text{ m}^2$$

and



## **FIGURE 4.14**

Example 4.7.

 $({}_{1}W_{2})_{\text{surface}} = -(0.0400 \text{ N/m})(0.0157 - 0 \text{ m}^{2})$ =  $-6.28 \times 10^{-4} \text{ N} \cdot \text{m} = -6.28 \times 10^{-4} \text{ J}$ =  $-(6.28 \times 10^{-4} \text{ J})(1 \text{ Btu}/1055 \text{ J}) = -5.96 \times 10^{-7} \text{ Btu}$ 

Table 4.2 Generalized Forces and Generalized Displacements		
Work Mode	Generalized Force F	Generalized Displacement $d_{\chi}$
Moving system boundary	p (pressure)	d₩ (volume)
Shaft	T (torque)	d heta (angular displacement)
Elastic	$-\sigma$ (stress)	$\forall d\epsilon$ (volume)
Surface tension $-\sigma_s$ (surface tension) $dA$ (surface area)		

Example 4.7 shows that it would take all of the surface tension energy stored in nearly 2 million 5 cm diameter soap bubbles to raise the temperature of one pound-mass of water by one degree Fahrenheit.

Notice that, in each of the four cases of classical mechanical work, the work differential  $\overline{d}W$  was given by the product of what we can call a *generalized force* F and a *generalized displacement*  $d\chi$ ; that is,

$$\overline{d}W = Fd\chi \tag{4.44}$$

where F and  $d\chi$  for each of the four classical mechanical work modes are identified in Table 4.2. In Eq. (4.44), the scalar or dot product is implied if F and  $d\chi$  are vectors.

The application of these work modes may change the thermodynamic state of the system and thus may produce a change in the system's thermodynamic properties. Finally, note that the generalized forces are all intensive properties, whereas the generalized displacements are all extensive properties.

We can generalize the work concept to nonmechanical systems by including any work mode given by Eq. (4.44) when the generalized force *F* is an intensive property *forcing function* and the generalized displacement  $d\chi$  is an extensive property *response function*. We are now in a position to analyze the remaining work mode energy transport mechanisms.

# 4.7 NONMECHANICAL WORK MODES OF ENERGY TRANSPORT

Of the wide variety of nonmechanical work modes available, the following five are of significant engineering value:

- **1.** Electrical current flow.
- **2.** Electrical polarization.
- **3.** Magnetic.
- 4. Chemical.
- 5. Mechanochemical.

Materials are electrically classified as conductors, nonconductors (dielectrics or insulators), and semiconductors. A pure *conductor* is a substance that has mobile charges (electrons) free to move in an applied electric field. They constitute the flow of electrical current. Pure *nonconductors* have no free electrons whatsoever, and a *semiconductor* is a material that behaves as a dielectric (nonconductor) at low temperatures but becomes conducting at higher temperatures.

As an electric field E is applied to a pure conductor, the free electrons migrate to the conductor's outer surface, where they create their own electric field, which opposes the applied field. As more and more electrons reach the outer surface, the electric field inside the object grows weaker and weaker, eventually vanishing altogether. At equilibrium, there is no electric field within a pure conductor.

A pure nonconductor has no free electrons with which to neutralize the applied electric field. The externally applied field therefore acts on the internal molecules, and normally nonpolar molecules become polar and develop *electric dipoles*. Some molecules are naturally polar in the absence of an electric field (e.g., water). The applied electric field rotates and aligns the newly created or naturally polar molecules. Complete alignment is normally prevented by molecular vibrations. But, when the applied field is strong enough to overcome the vibration randomizing effects and further increases in field strength have no effect on the material, the material is said to be *saturated* by the applied field. The process of electric dipole creation, rotation, and alignment in an applied electric field is known as dielectric *polarization*.

Therefore, two work modes arise from the application of an electric field to a material. The first is the work associated with the free electron (current) flow, and the second is the work associated with dielectric polarization. For a pure conductor, the polarization work is always zero; and for a pure nonconductor, the current flow work is always zero. We always treat these as separate work modes.

# **4.7.1 Electrical Current Flow Work**

Electrical current flow work occurs whenever current-carrying wires (pure conductors) cross the system boundary. This is the most common type of nonmechanical work mode encountered in thermodynamic system analysis. The generalized force here is the intensive property *voltage* (the electric potential)  $\phi$ , and the extensive property generalized displacement is the charge *q*.<sup>7</sup> Then, assuming the voltage is applied to the system,

$$(\overline{d}W)_{\text{electrical}} = -\phi dq$$

and

 $\binom{1}{1}W_{2}_{\text{current}} = -\int_{1}^{2} \phi \, dq \tag{4.45}$ 

Electrical current i is defined as

 $i = \frac{dq}{dt}$ 

so dq = i dt, and

$$\left(\overline{d}W\right)_{\substack{\text{electrical}\\ \text{current}}} = -\phi i \, dt \tag{4.46}$$

Then, electric current work is

## Electrical current work

$$\binom{1}{1}W_2 = \int_1^2 \phi i \, dt \tag{4.47}$$

From Ohm's law, the instantaneous voltage  $\phi$  across a pure resistance *R* carrying an alternating current, described by  $i = i_{\text{max}} \sin(2\pi f t)$ , is

$$\phi = Ri = Ri_{\max}\sin\left(2\pi ft\right)$$

where *f* is the frequency and  $\phi_{\text{max}} = Ri_{\text{max}}$ . Thus, Eq. (4.47) gives the electrical current work of *n* cycles of an alternating electrical current applied to a pure resistance from time 0 to time t = n/f as

$$({}_{1}W_{2})_{\text{electrical}} = -\phi_{\max}i_{\max}\int_{0}^{t=n/f} \sin^{2}(2\pi ft) dt$$

$$= -\phi_{\max}i_{\max}(t/2)$$

$$= -\phi_{e}i_{e}t = -\phi_{e}^{2}(t/R) = -i_{e}^{2}Rt$$
(4.48)

where  $\phi_e$  and  $i_e$  are the *effective* voltage and current defined by  $\phi_e = \phi_{max}/\sqrt{2}$  and  $i_e = i_{max}/\sqrt{2}$ .

Electrical work can exist in either open or closed systems (we do not consider the flow of electrons across a system boundary to be a mass flow term). When the electron supply is going into a *finite* system, such as a battery or a capacitor, Eq. (4.45) or (4.47) is convenient to use. But, when an essentially *infinite* supply of voltage and current is used, it is more convenient to use the instantaneous rate at which electrical work is done, or the electrical *power*, defined as

$$(\dot{W})_{\text{electrical}} = \frac{\overline{d}W}{dt} = -\phi i \tag{4.49}$$

# OHM'S LAW

This law was discovered experimentally by George Simon Ohm (1787–1854) in 1826. Basically, it states that, for a given conductor, the current is directly proportional to the potential difference, usually written as  $\phi = Ri$ , where *R* is the *electrical resistance* in units of ohms, where 1 ohm = 1 volt/ampere.

<sup>&</sup>lt;sup>7</sup> The electrical potential  $\phi$  and the electric field strength vector *E* are related by  $E = -\nabla(\phi)$ , where  $\nabla(\phi)$  is the gradient operator.

The instantaneous electrical power  $-\phi i$  of an alternating current circuit varies in time with the excitation frequency *f*. However, it is common to report the electrical power of an ac device as the instantaneous power averaged over one cycle of oscillation, or

$$(\dot{W})_{\text{electrical}}_{(\text{pure resistance})} = -f \int_{0}^{1/f} \phi i \, dt = -f \phi_{\text{max}} i_{\text{max}} \int_{0}^{1/f} \sin^{2}(2\pi ft) \, dt$$

$$= -\phi_{\text{max}} i_{\text{max}}/2 = -\phi_{e} i_{e} = -\phi_{e}^{2}/R = -i_{e}^{2}R$$
(4.50)

where  $\phi_e$  and  $i_e$  are the effective voltage and current defined earlier.

# **EXAMPLE 4.8**

Consider the 120. V, 144  $\Omega$  (ohm), alternating current incandescent lightbulb shown in Figure 4.15 to be a pure resistance. Determine

- **a.** The electrical current work when the bulb is operated for 1.50 h.
- **b.** Its electrical power consumption.

#### Solution

**a.** Since the voltage and current ratings of ac devices are always given in terms of their effective values,  $\varphi_e = 120$ . *V* and, from Ohm's law,  $i_e = \varphi_e/R = 120./144 = 0.833$  A. Then, from Eq. (4.48),

$$({}_{1}W_{2})_{\text{electrical}} = -\phi_{e}i_{e}t = -(120. \text{ V})(0.833 \text{ A})(1.50 \text{ h})$$
  
current  
 $= -150. \text{ V} \cdot \text{A} \cdot \text{h} = -150. \text{ W} \cdot \text{h}$ 

**b.** From Eq. (4.50),

$$(\dot{W})_{\text{electrical}} = -\phi_e i_e = -(120.\,\text{V})(0.833\,\text{A}) = -100.\,\text{V}\cdot\text{A} = -100.\,\text{W}$$
  
current

The minus signs appear because electrical work and power go into the system.

#### Exercises

- **16.** Determine the work and power consumption in Example 4.8 when the bulb is operated for 8.00 h instead of 1.50 h. **Answer:**  $(_1W_2)_{\text{electrical}} = -800$ . W·h, and  $\dot{W}_{\text{electrical}} = -100$ . W.
- **17.** Determine the effective current drawn by a 1.00 hp ac electric motor operating on a standard 120. V effective power line. **Answer**:  $i_e = 6.22$  A.
- **18**. Determine the electrical power dissipated by an 8-bit microprocessor computer chip that draws 90.0 mA at 5.00 V dc. **Answer**:  $\dot{W}_{\text{electrical}} = -450$ . mW.

## **4.7.2 Electrical Polarization Work**

The electric dipole formation, rotation, and alignment that occur when an electric field is applied to a nonconductor or a semiconductor constitutes an electric polarization work mode. The generalized force is the intensive property  $\vec{E}$  (in V/m), the electric field strength vector, and the generalized displacement is the extensive property  $\vec{P}$  (in A·s/m<sup>2</sup>), the polarization vector of the medium (defined to be the sum of the electric dipole rotation moments of all the molecules in the system). Then, assuming the electric field is applied *to* the system,

$$(\vec{d}W)_{\substack{\text{electrical}\\\text{polarization}}} = -\vec{E} \cdot \vec{dP}$$
(4.51)

and

$$\binom{1}{1}W_{2}_{\substack{\text{electrical}\\\text{polarization}}} = -\int_{1}^{2} \vec{E} \cdot d\vec{P}$$
(4.52)

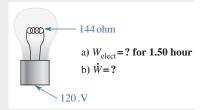


FIGURE 4.15 Example 4.8.

Material	Temperature (°C/°F)	$\chi_e$ (dimensionless)
Air (14.7 psia)	20/68	$5.36 \times 10^{-4}$
Plexiglass	27/81	2.40
Neoprene rubber	24/75	5.7
Glycerine	25/77	41.5
Water	25/77	77.5

Thermodynamics, second ed. McGraw-Hill, New York.

Since the effect of the electric field is to orient the dipoles coincident with the field, then  $\vec{E}$  and  $\vec{P}$  are always parallel and point in the same direction. Therefore, if we let the magnitude of  $\vec{E}$  be *E* and the magnitude of  $\vec{P}$  be *P*, then Eqs. (4.51) and (4.52) reduce to

$$(\overline{d}W)_{\substack{\text{electrical}\\\text{polarization}}} = -E\,dP \tag{4.53}$$

and

$$\binom{W_2}{\text{polarization}} = -\int_1^2 E \, dP \tag{4.54}$$

Many substances (particularly gases) correlate well with the following dielectric equation of state:

$$P = \varepsilon_0 \chi_e \Psi E \tag{4.55}$$

where  $\forall$  is the volume of the dielectric substance,  $\varepsilon_0$  is the electric permittivity of vacuum (8.85419 × 10<sup>-12</sup> N/V<sup>2</sup>), and  $\chi_e$  is the *electric susceptibility* (a dimensionless number) of the material. Table 4.3 gives values of  $\chi_e$  for various materials.

# **EXAMPLE 4.9**

The parallel plate capacitor shown in Figure 4.16 is charged to a potential difference of 120. V at 25.0°C. The plates are square with a side length of 0.100 m and are separated by 0.0100 m. If the gap between the plates is filled with water, determine the polarization work required in the charging of the capacitor.

#### Solution

Here, we can use the dielectric equation of state, Eq. (4.55). Then, Eq. (4.54) becomes

$$({}_{1}W_{2})_{\text{electric}}_{\text{polarization}} = -\int_{1}^{2} E \, dP = -\int_{1}^{2} (\varepsilon_{0} \chi_{e} \Psi E) \, dE = -\varepsilon_{0} \chi_{e} \Psi (E_{2}^{2} - E_{1}^{2})/2$$

From the problem statement, we have

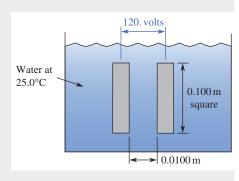
$$\Psi = AL = (0.100 \text{ m})^2 (0.0100 \text{ m}) = 1.00 \times 10^{-4} \text{ m}^3$$

If we assume that the electrical potential  $\boldsymbol{\varphi}$  varies linearly between the plates, then we can write

$$E = |-\nabla(\phi)| = (\text{voltage difference})/(\text{plate gap}) \text{ with } E_1 = 0 (\text{uncharged plates})$$

and

$$E_2 = \frac{120.\text{V}}{0.0100 \text{ m}} = 1.20 \times 10^4 \text{ V/m} \text{ (charged plates)}$$





(Continued)

## **EXAMPLE 4.9** (Continued)

From Table 4.3, we find that, for water,  $\chi_e = 77.5$ . Then,

 $\binom{({}_{1}W_{2})_{\text{electric}}}{\text{polarization}} = -(8.85419 \times 10^{-12} \text{ N/V}^{2})(77.5)(1.00 \times 10^{-4} \text{m}^{3}) \times [(1.20 \times 10^{4})^{2} - 0^{2} \text{V}^{2}/\text{m}^{2}]/2$  $= -4.94 \times 10^{-6} \text{ N} \cdot \text{m} = -4.94 \times 10^{-6} \text{ J}$ 

The work is negative since it went *into* the capacitor (the system).

#### Exercises

- **19.** How much voltage would be required to store 1.00 MJ of electrical polarization work in the capacitor of Example 4.9? **Answer**:  $V = 3.82 \times 10^7$  V.
- **20.** Determine the electrical polarization work in Example 4.9 when the gap between the capacitor plates is filled with air at 20.0°C. Answer:  $({}_{1}W_{2})_{\text{polarization}} = -3.42 \times 10^{-11} \text{ J.}$
- **21.** A capacitor is made from two concentric cylinders 0.100 m long. The diameter of the outer cylinder is 0.0200 m and the diameter of the inner cylinder is 0.0100 m. The gap between the cylinders is filled with glycerine at 25.0°C. Determine the electrical polarization work required to charge the capacitor when 120. V is applied. **Answer**:  $(_1W_2)_{\text{polarization}} = -1.04 \times 10^{-10} \text{ J}$ .

The polarization work is a small fraction of the total energy required to charge an entire capacitor. The total work required to charge a capacitor is divided into two parts. The largest fraction goes into increasing the electric field strength  $\vec{E}$  itself, and the remaining goes into the polarization of the material exposed to the electric field. Consequently, if the thermodynamic system you are analyzing is just the material between the plates of a capacitor, then the only polarization work is done on the material and Eq. (4.54) gives the correct electrical work mode value. On the other hand, if you are analyzing the entire capacitor (plates and dielectric), then Eq. (4.47) must be used to determine the correct electrical work mode value.

## 4.7.3 Magnetic Work

Materials are classified as either diamagnetic, paramagnetic, or ferromagnetic. Diamagnetic materials have no permanently established molecular magnetic dipoles. However, when they are placed in a magnetic field, their molecules develop magnetic dipoles whose magnetic field opposes the applied field (the Greek prefix *dia* means "to oppose"). Paramagnetic materials have naturally occurring molecular magnetic dipoles. When placed in a magnetic field, these dipoles tend to align themselves parallel to the field (the Greek prefix *para* means "beside"). Ferromagnetic materials retain some magnetism after the removal of a magnetic field. The thermodynamic state of these materials depends not only on the present values of their thermomagnetic properties, but also on their magnetic history. In this sense, ferromagnetic materials have a "memory" of their previous magnetic exposure.

As in the case of an electric field, the work associated with the initiation or destruction of a magnetic field consists of two parts. The first part is the work required to change the magnetic field itself (as though it existed within a vacuum), and the second part is the work required to change the magnetization of the material present inside the magnetic field.

For calculating the total work of magnetization, the generalized force is the intensive property  $\vec{H}$  (in A/m<sup>2</sup>), the magnetic field strength, and the generalized displacement is the extensive property  $\forall \vec{B}$ , the product of the system volume  $\forall$  (in m<sup>3</sup>) and the magnetic induction  $\vec{B}$  (in tesla or V s/m<sup>2</sup>). Thus, assuming the magnetic field is applied to the system,

$$(\overline{d}W)_{\text{magnetic}} = -\overline{H} \cdot d(\overline{\forall} \, \overline{B})$$
(4.56)

and since  $\vec{H}$  and  $\vec{B}$  are always parallel and point in the same direction in magnetic materials, this reduces to

$$(\overline{d}W)_{\text{magnetic}} = -H \cdot d(\Psi B) \tag{4.57}$$

where *H* is the magnitude of  $\vec{H}$  and *B* is the magnitude of  $\vec{B}$ . The magnetic induction can be decomposed into two vectors as

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} \tag{4.58}$$

where  $\vec{M}$  is the magnetization vector per unit volume of material exposed to the magnetic field (in a vacuum,  $\vec{M}$  is equal to the null vector  $\vec{0}$ ), and  $\mu_0 = 4\pi \times 10^{-7} \text{ V} \cdot \text{s}/(\text{A} \cdot \text{m})$  is a universal constant called the *magnetic permeability*. Inserting this information into Eq. (4.57) gives

$$(\overline{d}W)_{\text{magnetic}} = -\mu_0 H d(\forall H) - \mu_0 H d(\forall M)$$
(4.59)
(4.59)

Equation (4.59) is the differential of the total work associated with changing a material's magnetic field. The first term corresponds to the work required just to change the field itself (in a vacuum); and the second term corresponds to the work associated with the alignment of the molecular magnetic dipoles of the material present inside the magnetic field and represents the work of magnetization of the material exposed to the magnetic field. Hence, we can write

$$(\overline{d}W)_{\text{material}} = -\mu_0 H d(\Psi M)$$
(4.60)

A simple and useful equation of state for a magnetic field is

$$M = \chi_m H \tag{4.61}$$

where  $\chi_m$  is the *magnetic susceptibility* (a dimensionless number) of the material. The magnetic susceptibility is negative for diamagnetic materials and positive for paramagnetic materials (see Table 4.4). For a constant volume magnetization process, Eq. (4.61) can be used in Eq. (4.59) to give

$$(\overline{d}W)_{\text{magnetic}} = -\mu_0 \Psi(1+\chi_m) H dH$$

and assuming a constant volume and a constant magnetic susceptibility, this can be integrated to give the total magnetic work:

#### Total magnetic work

$$\binom{1}{1} \binom{W_2}{\max_{\text{(total)}}} = -\mu_0 \Psi(1+\chi_m) \left(\frac{H_2^2 - H_1^2}{2}\right)$$
(4.62)

where the increment to the total work due to the actual magnetization of the exposed material is just the actual magnetic work:

## Actual magnetic work

$$\binom{1}{1} \binom{1}{1} \binom{1}{2} m_{\text{agnetization}} = -\mu_0 \frac{1}{2} \frac{1}{2} \binom{H_2^2 - H_1^2}{2}$$
(4.63)

Table 4.5 summarizes the electrical and magnetic symbols used in this section.

Thermodynamics, second ed. McGraw-Hill, New York.

Material	Temperature (°C/°F)	$\chi_m$ (dimensionless)
Mercury	18/26	$-3.2 \times 10^{-5}$
Quartz	25/77	$-1.65 \times 10^{-5}$
Ice	0/32	$-0.805 \times 10^{-5}$
Nitrogen (14.7 psia)	20/68	$-0.0005 \times 10^{-5}$
Oxygen (14.7 psia)	20/68	$0.177 \times 10^{-5}$
Aluminum	18/64	$2.21 \times 10^{-5}$
Platinum	18/64	$29.7 \times 10^{-5}$

Table 4.5         Summary of Electrical and Magnetic Terms			
Symbol	Name	SI Units	
E P ε <sub>0</sub> χ <sub>e</sub> H B M	Electric field strength Polarization Permittivity of free space Electric susceptibility Magnetic field strength Magnetic induction Magnetization	V/m A $\cdot$ s/m <sup>2</sup> 8.85419 × 10 <sup>-12</sup> N/V <sup>2</sup> Dimensionless A/m Tesla or V $\cdot$ s/m <sup>2</sup> A/m	
$\mu_0$	Magnetic permeability	$4\pi \times 10^{-7} \text{ V} \cdot \text{s/A} \cdot \text{m}$	

# **EXAMPLE 4.10**

The magnetic susceptibility of the diamond in the gold engagement ring shown in Figure 4.17 is  $-2.20 \times 10^{-5}$  at 20.0°C. Determine the (a) total magnetic and (b) material magnetic work required to change the magnetic field of a 1 carat diamond having a volume of  $5.00 \times 10^{-6}$  m<sup>3</sup> from 0.00 to  $1.00 \times 10^{3}$  A/m.

#### Solution

a. The total magnetic work required is given by Eq. (4.62) as

$$({}_{1}W_{2})_{\text{magnetic}} = -\mu_{0} \Psi (1 + \chi_{m}) \left(\frac{H_{2}^{2} - H_{1}^{2}}{2}\right)$$

a) W<sub>total magnetic</sub>=? b) W<sub>material magnetic</sub>=? 1 carat diamond

**FIGURE 4.17** 

Example 4.10.

where  $\mu_0 = 4\pi \times 10^{-7} \text{ V} \cdot \text{s/A} \cdot \text{m}$  and  $\chi_m = -2.20 \times 10^{-5}$ . Then,

$$({}_{1}W_{2})_{\text{magnetic}} = -\left(4\pi \times 10^{-7} \frac{\text{V} \cdot \text{s}}{\text{A} \cdot \text{m}}\right)(5.00 \times 10^{-6} \text{ m}^{3})(1 - 2.20 \times 10^{-5})\left(\frac{1.00 \times 10^{6} - 0 \text{ A}^{2}/\text{m}^{2}}{2}\right)$$
$$= -3.14 \times 10^{-6} \text{ J}$$

b. The magnetic work required to change the magnetic field strength inside the diamond alone is given by Eq. (4.63) as

$$({}_{1}W_{2})_{\text{material}}_{\text{magnetization}} = -\mu_{0} \forall \chi_{m} \left(\frac{H_{2}^{2} - H_{1}^{2}}{2}\right)$$

and, using the values from part a, we get

$$\binom{2W^2}{magnetic} = -\left(4\pi \times 10^{-7} \frac{V \cdot s}{A \cdot m}\right) (5.00 \times 10^{-6} \text{ m}^3) (-2.20 \times 10^{-5}) \left(\frac{1.00 \times 10^6 - 0 \text{ A}^2/\text{m}^2}{2}\right)$$
  
= 6.91 × 10<sup>-11</sup> J

#### Exercises

- **22.** The magnetic susceptibility of gold is  $-3.60 \times 10^{-5}$ . If the gold in the ring of Example 4.10 has a volume of  $1.00 \times 10^{-5}$  m<sup>3</sup>, determine the total magnetic work required to change the magnetic field strength of the ring (the gold plus the diamond) from 0 to  $1.00 \times 10^{3}$  A/m. Answer:  $(_{1}W_{2})_{\text{magnetic}} = -9.42 \times 10^{-6}$  J.
- **23.** The magnetic susceptibility of a ferromagnetic material such as iron varies with the applied magnetic field. However, if we assume it is constant over a small range of field strength at a value of 1800, then determine the (a) total work and (b) the material work required to magnetize a rectangular iron bar 0.500 inches square by 6.00 inches long from an initial magnetic field strength of zero to a magnetic field strength of 100. A/m. **Answer:**  $({}_{1}W_{2})_{total} = ({}_{1}W_{2})_{tion} = -2.78 \times 10^{-4}$  J.

## 4.7.4 Chemical Work

Chemical work occurs whenever a specific chemical species is added to or removed from a system. Here, the generalized force is the intensive property  $\mu_{i\nu}$  the Gibbs chemical potential of chemical species *i*, and the generalized displacement is the extensive property  $m_{i\nu}$  the mass of the chemical species added or removed.<sup>8</sup> Since any number of chemical species may be involved in a process, we write the chemical work as the sum over all *k* of the *i* species that are moved from the system to the surroundings as

$$(\overline{d}W)_{\text{chemical}} = -\sum_{i=1}^{k} \mu_i \, dm_i \tag{4.64}$$

and so

$$({}_{1}W_{2})_{\text{chemical}} = -\int_{1}^{2} \sum_{i=1}^{k} \mu_{i} dm_{i}$$
(4.65)

<sup>&</sup>lt;sup>8</sup> In chemistry texts, the chemical potential is usually defined on a molar (i.e., per unit gram mole) basis. In this text, we define it as a standard intensive (per unit mass) property.

When the chemical potential is constant during the mass transfer from state 1 to state 2, Eq. (4.65) can be integrated to give the chemical work of adding chemical species:

Chemical work of adding chemical species

$$\binom{W_2}{\mu_i = \text{constant}} = -\sum_{i=1}^{\kappa} \mu_i (m_2 - m_1)_i$$
(4.66)

Chemical work does not include the energy transports produced by chemical reactions, nor does it include the energy transported across the system boundary with the mass transport itself. Mass flow energy transport is considered later in this chapter, and the energy transports of chemical reactions are studied in detail in Chapter 9. The chemical work presented here essentially deals only with those energy transports involved in the mixing or separating of chemical species.

## **4.7.5 Mechanochemical Work**

Mechanochemical work occurs whenever there is a direct energy conversion from chemical to mechanical energy. Animal muscles are examples of mechanochemical systems. Small mechanochemical engines have also been built using this work mode, and Figure 4.18 shows a small hydraulic pump driven by a mechanochemical contractile fiber. The "fuel" used in mechanochemical engines is not "burned," as in a standard heat engine. Often it is merely diluted and a small amount of chemical work is simultaneously extracted.

Mechanochemical work is calculated as basic mechanical work. The generalized force is the intensive property f, the force generated by or *within* the mechanochemical system, and the generalized displacement is the extensive property  $\ell$ , the mechanical displacement of the system. Therefore,

$$(\bar{d}W)_{\text{mechanochemical}} = f \, d\ell \tag{4.67}$$

Generally, the mechanochemical force f is not constant during the contraction-expansion cycle, so the total mechanochemical work must be determined by a careful integration:

Mechanochemical work

$$({}_{1}W_{2})_{\text{mechanochemical}} = \int_{1}^{2} f \, d\ell \tag{4.68}$$

Note that, since the mechanochemical force comes from inside the system, a negative sign is not needed in Eqs. (4.67) and (4.68).

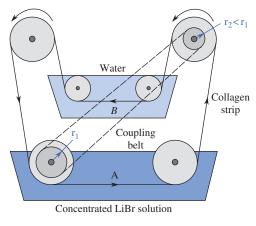
A system may be exposed to only one of these work modes of energy transport, or it may be exposed to several of them simultaneously. Since work is an additive quantity, to get the total (or net) work of a system that has more than one work mode present, we simply add all these work terms together:

Total differential work of all the work modes present

$$(\overline{d}W)_{\text{total}} = p \, d\Psi + T \cdot d\theta - \sigma \, d\varepsilon - \sigma_s \, dA$$

$$- \phi i \, dt - E \, dP - \mu_0 H \, d(\Psi M) - \sum_{i=1}^k \mu_i \, dm_i + f d\ell + \cdots$$
(4.69)

It is generally the engineer's responsibility to determine the number and type of work modes present in any problem statement or real world situation. Often, the work modes of a problem are affected by how the system boundaries are drawn (recall that boundary definition is a prerogative of the problem solver). For example, if a system contains an electrical heater, then electrical current work is done on the system. However, if the boundary is drawn to exclude the heating element itself, then no electrical work occurs and the energy transport becomes a heat transport from the surface of the heating element into the system.



#### **FIGURE 4.18**

A simple mechanochemical Katchalsky engine.

# **4.8 POWER MODES OF ENERGY TRANSPORT**

In thermodynamics, the time rate of change of a work mode,  $\overline{dW}/dt$ , is called *power*, and it represents the *power* mode of an energy transport  $\dot{W}$ . Dividing each of the previous nine differential work mode equations by the time differential dt produces an equation for the associated power mode. These results, summarized in Table 4.6, are useful in calculating the power (i.e., work rates) in problems in which continuous rate processes occur. While continuous rate processes can occur in both closed and open systems, they are more common in open systems.

# 4.9 WORK EFFICIENCY

Notice that, in all the work mode formulae given so far, no mention was made of the *efficiency* of the work transport of energy. This is because all the mechanical and nonmechanical work mode formulae discussed earlier were developed under the presumption of ideal circumstances, in which there were no friction losses or other inefficiencies within the system. Under these conditions the work process could ideally be reversed at any time, and all the work put into a system could be removed again simply by reversing the direction of the generalized force. Therefore, we call all the mechanical and nonmechanical work (or power) mode formulae developed previously *reversible* work (or power) formulae. Consequently—and this is very important—work or power calculations made with these formulae do not agree with the measurement of *actual* work that occurs in a real system. In real systems that absorb work, *more* actual work than that calculated from the previous formulae are required to produce the same effect on the system, and in real work producing systems, *less* actual work is produced than calculated from the previous formulae.

In the real world, nothing is reversible. Not one of the work modes discussed earlier can actually be carried out with 100% efficiency. Some are very close to being reversible (i.e., they have very high efficiencies) but none is completely reversible. This lack of reversibility in the real world is due to a phenomenon of nature that we describe with the second law of thermodynamics, which is discussed in detail in Chapter 7. Work modes with a low degree of reversibility (i.e., high irreversibility) are those carried out with systems far from thermodynamic equilibrium. Heat transfer, rapid chemical reactions (explosions), mechanical friction, and electrical resistance are all common sources of irreversibility in engineering systems.

Engineers use the concept of a work transport energy conversion efficiency to describe the difference between reversible and actual work. A general definition of the concept of an energy conversion efficiency is

Enormy conversion officiency $-n$	Desired energy result	(4.70)
Energy conversion efficiency = $\eta_E$ =	Required energy input	(4.70)

Table 4.6 Power Modes of Energy Transport			
Work Mode	Power Equation		
Mechanical moving boundary	$(\dot{W})_{\text{moving}} = p \frac{d\Psi}{dt} = p \dot{\Psi}$		
Mechanical rotating shaft	$(\dot{W})_{\text{rotating}} = T\left(\frac{d}{dt}\right) = T\omega$		
Mechanical elastic	$(\dot{W})_{\text{elastic}} = -\sigma \forall \left(\frac{d\epsilon}{dt}\right) = -\sigma \forall \dot{\epsilon}$		
Mechanical surface tension	$(\dot{W})_{\text{surface}} = -\sigma_s \left(\frac{dA}{dt}\right) = -\sigma_s \dot{A}$		
Electrical current	$(\dot{W})_{\text{electrical}} = -\phi i$		
Electrical polarization	$(\dot{W})_{\text{electrical}} = -E\left(\frac{dP}{dt}\right) = -E\dot{P}$		
Magnetic	$ (\dot{W})_{\text{magnetic}} = -\mu_0 \forall (1+\chi_m) H \left( \frac{dH}{dt} \right) $ = $-\mu_0 \forall (1+\chi_m) H \dot{H} $		
Chemical	$(\dot{W})_{\text{chemical}} = -\sum \mu_i \left(\frac{dm_i}{dt}\right) = -\sum \mu_i \dot{m}_i$		
Mechanochemical	$(\dot{W})_{\text{mechanochemical}} = f\left(\frac{d\ell}{dt}\right) = f\dot{\ell}$		

In the case of work-absorbing systems, such as pumps or compressors, we can use an equation similar to Eq. (4.70) to define a *work transport energy conversion efficiency*, or *reversible efficiency*,  $\eta_W$ , as work efficiency for work-absorbing systems:

Work efficiency for work-absorbing systems

$$\eta_W(\%) = \frac{W_{\text{rev}}}{W_{\text{act}}} \times 100 = \frac{\dot{W}_{\text{rev}}}{\dot{W}_{\text{act}}} \times 100$$
(4.71)

In the case of work-producing systems, such as engines or electrical generators, the reversible or work transport energy conversion efficiency becomes:

$$\eta_W(\%) = \frac{W_{\text{act}}}{W_{\text{rev}}} \times 100 = \frac{W_{\text{act}}}{\dot{W}_{\text{rev}}} \times 100$$
(4.72)

When these systems consist only of mechanical components, as, for example, in an internal combustion engine, the work transport energy conversion efficiency is simply called the *mechanical efficiency* and  $\eta_W$  is usually written as  $\eta_m$ .

Even though work transport energy conversion efficiencies are always less than 100%, not all energy conversion efficiencies are less than 100%. The value of the efficiency depends on the nature of the desired result in Eq. (4.70). An electrical resistance can convert electrical energy (the energy input) into heat (the desired result) with an energy conversion efficiency of 100%, but when this process is reversed, we find that the conversion of heat into work occurs with a much lower efficiency (a consequence of the second law of thermodynamics). On the other hand, refrigeration systems normally produce more "desired result" (cooling) than it actually costs in required energy input. Such systems normally have energy conversion efficiencies far in excess of 100%, not because they violate any law of physics, but simply because of the way their energy conversion efficiency is defined. Because it seems paradoxical to most people to speak of efficiencies in excess of 100%, we call such efficiencies *coefficients of performance* (COPs) instead. For example,

$$(COP)_{refrigerator} = \frac{Refrigerator cooling rate}{Refrigerator power input}$$

# EXAMPLE 4.11

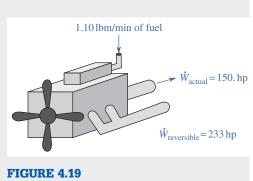
The automobile engine shown in Figure 4.19 produces 150. hp on a test stand while consuming fuel with a heat content of  $20.0 \times 10^3$  Btu/lbm at a rate of 1.10 lbm/min. A design engineer calculates the reversible power output from the engine as 223 hp. Determine

- a. The energy conversion efficiency of the engine.
- **b.** The work efficiency of the engine.

#### Solution

a. The energy conversion efficiency is given by Eq. (4.70) as

$$\eta_E = \frac{\text{Desired energy result}}{\text{Required energy input}}$$





The desired energy result here is the engine output power, 150. hp. The required energy input here is the energy coming from the fuel,  $20.0 \times 10^3$  Btu/lbm  $\times 1.10$  lbm/min  $\times 60$  min/h =  $1320 \times 10^3$  Btu/h  $\times (1 \text{ hp})/(2545 \text{ Btu/h}) = 519$  hp. Then,

$$\eta_{\rm E} = \frac{150.\,\rm hp}{519\,\rm hp} = 0.289 = 28.9\%$$

b. Since an engine is a work producing machine, Eq. (4.72) gives the work efficiency as

$$\eta_W = \frac{\dot{W}_{\text{actual}}}{\dot{W}_{\text{reversible}}} \times 100 = \frac{150.\,\text{hp}}{223\,\text{hp}} \times 100 = 67.3\%$$

(Continued)

# **EXAMPLE 4.11** (Continued)

#### Exercises

- 24. If the energy conversion efficiency in Example 4.11 were 15.5%, what would be the power output of the engine measured on the test stand for the same fuel flow rate? Answer:  $\dot{W}_{actual} = 80.4$  hp.
- **25.** An engineer designs a pump that requires 1.30 kW of reversible power to operate. A prototype pump is made and taken to the test laboratory. The actual power required to operate the prototype pump is measured at 1.50 kW. Determine the work (or mechanical) efficiency of this pump. **Answer**:  $\eta_W = 86.7\%$ .
- **26.** A refrigeration system is powered by a 5.0 kW electric motor. It removes  $18 \times 10^3$  J/s from the cold storage space. What is the coefficient of performance of this refrigeration system? **Answer**: COP = 3.6.

Because of the many irreversibilities that occur within a system, we cannot calculate actual work absorbed or produced from a theoretical formula. All efficiency values are determined from laboratory or field measurements on the actual work of real operating systems. When energy conversion efficiencies are to be taken into account in textbook problems, the efficiency values usually are provided within the problem statement. Experienced engineers often have a "feel" for what the efficiencies of certain devices should be, and they can use these efficiency estimations in their design calculations. Student engineers, however, are not presumed to be innately blessed with this knowledge.

The general form of Eq. (4.70) allows the creation of many different types of efficiencies. There are thermal, mechanical, volumetric, thermodynamic, and total efficiencies (to name just a few) in today's engineering literature. One should always be sure to understand the type of efficiency being used in any calculation.

# 4.10 THE LOCAL EQUILIBRIUM POSTULATE

Surprisingly, there is no adequate definition for the thermodynamic properties of a system that is not in an equilibrium state. Some extension of classical equilibrium thermodynamics is necessary for us to be able to analyze nonequilibrium (or irreversible) processes. We do this by subdividing a nonequilibrium system into many small but finite volume elements, each of which is larger than the local molecular mean free path, so that the continuum hypothesis holds. We then assume that each of these small volume elements is in *local* equilibrium. Thus, a nonequilibrium system can be broken down into a very large number of very small systems, each of which is at a different equilibrium state. This technique is similar to the continuum hypothesis, wherein continuum equations are used to describe the results of the motion of discrete molecules (see Chapter 2).

The differential time quantity dt used in nonequilibrium thermodynamic analysis cannot be allowed to go to zero as in normal calculus. We require that  $dt > \sigma_s$ , where  $\sigma$  is the time it takes for one of the volume elements of the subdivided nonequilibrium system just described to "relax" from its current nonequilibrium state to an appropriate equilibrium state. This is analogous to not allowing the physical size of the element to be less than its local molecular mean free path, as required by the continuum hypothesis. The error incurred by these postulates is really quite small, because they are the result of second-order variations of the thermodynamic variables from their equilibrium values. However, just as the continuum hypothesis can be violated by systems such as rarefied gases, the local equilibrium postulate can also be violated by highly nonequilibrium systems such as explosive chemical reactions. In the case of such violations, the analysis must be carried forward with techniques of statistical thermodynamics.

Because of the similarity between the local equilibrium postulate and the continuum hypothesis, it is clear that the local equilibrium postulate could as well be called the *continuum thermodynamics hypothesis*.

## SIMPLE SYSTEM

Any two independent intensive property values are sufficient to determine (or "fix") the local equilibrium state of a simple system.

# 4.11 THE STATE POSTULATE

To carry out a reversible work mode calculation using the formulae given earlier, we must know the exact behavior of both the generalized force (an intensive property) and the generalized displacement (an extensive property) for each work mode. Systems with multiple work modes have a variety of property values that must be monitored during the work process to utilize the proper work mode formulae. Therefore, it seems reasonable to expect that a simple relation exists between the number of work modes present in any given system and the number of independent property values required to fix the state of that system. This is the purpose of the following *state postulate*:

The number of independent intensive thermodynamic property values required to fix the state of a closed system that is

- **1.** Subject to the conditions of local equilibrium,
- 2. Exposed to *n* (nonchemical) work modes of energy transport, and
- **3.** Composed of *m* pure substances is n + m.

Therefore, a pure substance (m = 1) subjected to only one work mode (n = 1) requires two (n + m = 2) independent property values to fix its state. Such systems are called *simple systems*, and any two independent intensive properties determine (or "fix") its state.

The compression or expansion of a pure gas or vapor is a simple system. The work mode is moving system boundary work, and any two independent intensive property values (p, v; p, T; v, T, etc.) fix its state. In fact, a simple system occurs when each of the nonchemical reversible work modes just discussed is individually applied to a pure substance. On the other hand, if two of them are simultaneously applied to a pure substance, then n + m = 3 and three independent intensive property values are required to fix the state of the system.

# 4.12 HEAT MODES OF ENERGY TRANSPORT

We now introduce the three basic modes of heat transport of energy. Since a good heat mode analysis is somewhat more complex than a work mode analysis and since its understanding is very important to a good engineering education, most mechanical engineering curricula include a separate heat transfer course on this subject. Consequently, this section is meant to be only an elementary introduction to this subject.

A system with no heat transfer is said to be *adiabatic*, and all well-insulated systems are considered to be adiabatic. A process that occurs with no heat transport of energy is called an *adiabatic process*.

In the late 18th century, heat was thought to be a colorless, odorless, and weightless fluid, then called *caloric*. By the middle of the 19th century, it had been determined that heat was in fact not a fluid but rather it represented energy in transit. Unfortunately, many of the early heat-fluid technical terms survived and are still in use today. This is why we speak of heat transfer and heat flow, as though heat were something physical, but it is not. Because these conventions are so deeply ingrained in our technical culture, we use the phrases *heat transfer, heat transport*, and the *heat transport of energy* interchangeably.

After it was determined that heat was not a fluid, late 19th century physicists defined heat transfer simply as energy transport due to a temperature difference. In this framework, temperature was the only intensive property driving force for the heat transport of energy.

Today, the simplest way to define heat transport of energy is as any energy transport that is neither a work mode nor a mass flow energy transport mode. More precisely, modern nonequilibrium thermodynamics defines heat transfer as just the transport of internal energy into or out of a system. With this definition, all other energy transport modes are automatically either work or mass flow modes.

The basic heat transfer formulae were developed empirically and, unlike the previous work mode formulae, give actual rather than reversible heat transport values. In fact, since heat transfer always occurs as a result of energy

# WHAT DOES THE WORD ADIABATIC MEAN?

The term *adiabatic* was coined in 1859 by the Scottish engineer William John Macquorn Rankine (1820–1872). It comes from the Greek word,  $\alpha \delta \alpha \beta \alpha \tau \sigma \sigma$ , meaning "not to pass through." In thermodynamics, it means heat does not pass through the system boundary, or simply that there is no heat transfer. *Adiabatic* is the analog of the word *aergonic* (meaning "no work") introduced earlier in this chapter.

spontaneously moving *down* a potential gradient (such as from high to a low temperature) and the reverse cannot spontaneously occur, no heat transfer process can be reversed in any way whatsoever. Therefore, all finite heat transfer processes are irreversible.

# **4.13 HEAT TRANSFER MODES**

Heat transfer is such a large and important mechanical engineering topic that most curricula have at least one required course in it. Heat transfer equations are always cast as heat transfer *rate* (i.e.,  $\dot{Q}$ ) equations. To determine the amount of heat energy transport that occurs as a system undergoes a process from one equilibrium

state to another you must integrate  $\dot{Q}$  over the time interval of the process, or  ${}_{1}Q_{2} = \int_{1}^{2} \dot{Q} dt$ . Normally, we

choose processes in which  $\dot{Q}$  is constant in time so that the integral becomes simply  ${}_{1}\dot{Q}_{2} = \dot{Q}(t_{2} - t_{1}) = \dot{Q}(\Delta t)$ , where  $\Delta t$  is the time required for the process to occur.

Historically, the field has been divided into three heat transfer modes: conduction, convection, and radiation. These three modes are briefly described next.

## 4.13.1 Conduction

The basic equation of conduction heat transfer is Fourier's law:

$$\dot{Q}_{\rm cond} = -k_t A \left(\frac{dT}{dx}\right) \tag{4.73}$$

where  $\dot{Q}_{cond}$  is the conduction heat transfer rate,  $k_t$  is the thermal conductivity of the material, A is the crosssectional area normal to the heat transfer direction, and dT/dx is the temperature gradient in the direction of heat transfer. The algebraic sign of this equation is such that a positive  $\dot{Q}_{cond}$  always corresponds to heat transfer in the positive x direction, and a negative  $\dot{Q}_{cond}$  always corresponds to heat transfer in a negative x direction. Since this is not the same sign convention adopted earlier in this text, the sign of the values calculated from Fourier's law may have to be altered to produce a positive when it enters a system and a negative when it leaves a system.

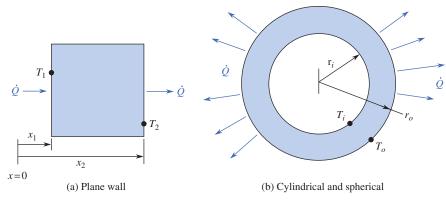
For steady conduction heat transfer through a plane wall (Figure 4.20), Fourier's law can be integrated to give

$$(\dot{Q}_{\text{cond}})_{\text{plane}} = -k_t A \left( \frac{T_2 - T_1}{x_2 - x_1} \right)$$
 (4.74)

and for steady conduction heat transfer through a hollow cylinder of length L, Fourier's law can be integrated to give

$$(\dot{Q}_{\text{cond}})_{\text{cylinder}} = -2\pi L k_t \left[ \frac{T_{\text{inside}} - T_{\text{outside}}}{\ln(r_{\text{inside}}/r_{\text{outside}})} \right]$$
(4.75)

Table 4.7 gives thermal conductivity values for various materials.





Thermal conduction notation in plane, cylindrical, and spherical coordinates.

Table 4.7 Thermal Conductivity of Various Materials				
	Thermal Conductivity k <sub>t</sub>			
Material	Temperature (°C/°F)	Btu/(h · ft · R)	W/(m·K)	
Air (14.7 psia)	27/81	0.015	0.026	
Hydrogen (14.7 psia)	27/81	0.105	0.182	
Saturated water vapor (14.7 psia)	100/212	0.014	0.024	
Saturated liquid water (14.7 psia)	0/32	0.343	0.594	
Engine oil	20/68	0.084	0.145	
Mercury	20/68	5.02	8.69	
Window glass	20/68	0.45	0.78	
Glass wool	20/68	0.022	0.038	
Aluminum (pure)	20/68	118.0	204.0	
Copper (pure)	20/68	223.0	386.0	
Carbon steel (1% carbon)	20/68	25.0	43.0	

## 4.13.2 Convection

Convective heat transfer occurs whenever an object is either hotter or colder than the surrounding fluid. The basic equation of convection heat transfer is Newton's law of cooling:

$$\dot{Q}_{\rm conv} = hA(T_{\infty} - T_s) \tag{4.76}$$

where  $Q_{conv}$  is the convection heat transfer rate, *h* is the convective heat transfer coefficient, *A* is the surface area of the object being cooled or heated,  $T_{\infty}$  is the bulk temperature of the surrounding fluid, and  $T_s$  is the surface temperature of the object. The algebraic sign of Newton's law of cooling has been chosen to be positive for  $T_{\infty} > T_s$  (i.e., for heat transfer into the object). This corresponds to our thermodynamic sign convention for heat transfer when the object is the system. The convective heat transfer coefficient *h* is always a positive, empirically determined value. Table 4.8 lists typical heat transfer coefficients.

## 4.13.3 Radiation

All electromagnetic radiation is classified as radiation heat transfer. Infrared, ultraviolet, visible light, radio and television waves, X rays, and so on are all forms of radiation heat transfer. The radiation heat transfer between two objects situated in a nonabsorbing or emitting medium is given by the Stefan-Boltzmann law:

$$\dot{Q}_{\rm rad} = F_{1-2}\varepsilon_1 A_1 \sigma (T_2^4 - T_1^4) \tag{4.77}$$

where  $\dot{Q}_{rad}$  is the radiation heat transfer rate,  $F_{1-2}$  is called the *view factor* between objects 1 and 2 (it describes how well object 1 "sees" object 2),  $\varepsilon_1$  is the dimensionless emissivity or absorptivity (the hotter object is said to *emit* energy while the colder object *absorbs* energy) of object 1,  $A_1$  is the surface area of object 1,  $\sigma$  is the Stefan-Boltzmann constant (5.69 × 10<sup>-8</sup> W/m<sup>2</sup>·K<sup>4</sup> or 0.1714 × 10<sup>-8</sup> Btu/h·ft<sup>2</sup>·R<sup>4</sup>), and  $T_1$  and  $T_2$  are the surface temperatures of the objects. A *black* object is defined to be any object whose emissivity is  $\varepsilon = 1.0$ . Table 4.9 lists some typical emissivity values. Also, if object 1 is completely enclosed by object 2, then  $F_{1-2} = 1.0$ . For a completely enclosed black object, the Stefan-Boltzmann law reduces to

$$(\dot{\mathbf{Q}}_{\text{rad}})_{\substack{\text{black}\\\text{enclosed}}} = A_1 \sigma (T_2^4 - T_1^4)$$
(4.78)

Table 4.8 Typical Values of the Convective Heat Transfer Coefficient				
	Convect	Convective Heat Transfer Coefficient h		
Type of Convection	Btu/(h · ft <sup>2</sup> · R)	W/(m <sup>2</sup> · k)		
Air, free convection	1–5	2.5–25		
Air, forced convection	2–100	10–500		
Liquids, forced convection	20–3000	100–15,000		
Boiling water	500-5000	2500–25,000		
Condensing water vapor	1000–20,000	5000-100,000		

Table 4.9 Typical Emissivity Values for Various Materials			
Material	Temperature (°C/°F)	Emissivity $\varepsilon$ (dimensionless)	
Aluminum	100/212	0.09	
Iron (oxidized)	100/212	0.74	
Iron (molten)	1650/3000	0.28	
Concrete	21/70	0.88	
Flat black paint	21/70	0.90	
Flat white paint	21/70	0.88	
Aluminum paint	21/70	0.39	
Water	0–100/32–212	0.96	

The sign convention in the Stefan-Boltzmann law has been chosen to be positive when  $T_2 > T_1$ ; therefore, the "system" should be object 1 to achieve the correct thermodynamic sign convention. Also note that this equation contains the temperature raised to the fourth power. This means that *absolute* temperature units must always be used.

# 4.14 A THERMODYNAMIC PROBLEM SOLVING TECHNIQUE

The previous 11 example problems have been relatively straightforward, mainly illustrating the use of specific energy and work mode equations. However, most thermodynamics problems are not so straightforward, and now we are ready to introduce a comprehensive thermodynamic problem solving technique that allows you to set up and solve even the most complex thermodynamics problems.

Thermodynamic problem statements sometimes have the appearance of being stories full of technical jargon, liberally sprinkled with numbers. All too often, your first instinct on being faced with such a situation is to calculate something—anything—because the act of calculation brings about the euphoria of apparent progress toward a solution. However, this approach is quickly stalled by the inability to reach the final answer, followed by long frustrating periods of shoe shuffling and window staring until either enlightenment, discouragement, or sleep occurs. This is definitely the wrong problem solving technique. A good technique must have definite starting and ending points, and it must contain clear and logical steps that carry you toward a solution.

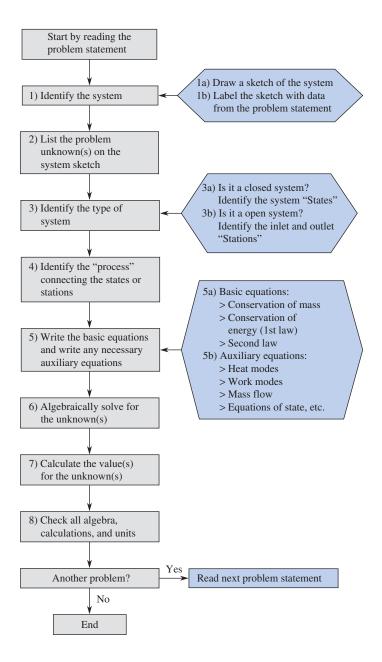
As a prelude to discussing the details of the problem solving technique, you should realize that the general structure of a thermodynamic word problem usually contains the following three features.

- 1. A thermodynamic problem statement is usually a small "story" that is too long to be completely and accurately memorized no matter how many times you read it. So simply reading the problem statement once is usually not enough; you must translate it into your own personal environment by adding a schematic drawing, writing down relevant assumptions, and beginning a structured solution.
- 2. To completely understand the problem statement, you must first "decode" it. That is, you must dissect and rearrange the problem statement until it fits into a familiar pattern. Any problem solving technique is, of course, based on the premise that the problem has a solution. Curiously, it is very easy to construct problem statements that are not solvable without the introduction of extraneous material (judiciously called *assumptions*).
- **3.** Thermodynamic problem statements tend to be very wide ranging. They can be written about virtually any type of system and can deal with virtually any form of technology. To give the problem statements a pragmatic engineering flavor, they are usually written as tiny stories that are designed to reflect what you will encounter as a working engineer.

Unfortunately, many students facing thermodynamics for the first time are overwhelmed by these factors. How are you supposed to know anything about how a nuclear power plant operates, how the combustion chamber of a turbojet engine functions, or how a boiler feed pump works if you have never actually seen one in operation? The key is that you really do not have to know that much about how these things work to carry out a good thermodynamic analysis of them. But, you do have to understand how problem statements are written and how to analyze them correctly. This is the core of the problem solving technique.

In fact, it would be possible to write a computer program that could solve any thermodynamic word problem. What we are going to do is to show you how to solve thermodynamic problems by using a computerlike flow-chart approach, as in Figure 4.21.

The technique is really very simple. First, you must learn to formulate a general starting point. Then you must learn to identify the key logical decisions that have to be made as the solution progresses. Finally, when all the



## FIGURE 4.21

Flowchart for solving thermodynamic problems.

analysis and algebraic manipulations are complete, you make the necessary calculations (paying close attention to units and significant figures) to obtain the desired results.

The steps to be followed are shown in Figure 4.21, and each step is discussed in detail next.

## Begin by carefully reading the problem statement completely through.

Step 1. Make a sketch of the system or device described in the problem statement and determine the material (air, steam, liquid water, etc.) with which you are working. Then, carefully define the part(s) you choose to analyze by inserting a dashed line to identify the system boundary.

Step 2. **Identify the problem's unknown(s)** by rereading the problem statement and picking out all the things you are supposed to determine. Write them on your system sketch.

Step 3. Determine whether it is a closed system or an open system. If your system is closed, identify as many of the state properties as you can. Most problems have only two states (initial and final), but some also have intermediate states with which you have to contend. To keep the numerical values and units of the state properties straight, list each one under a "state" heading.

## WHAT IS THE SECRET TO SOLVING THERMODYNAMICS PROBLEMS?

The secret to solving thermodynamic problems is to *do the analysis first* and *do the calculations last*, not the other way around. The basic process for solving a thermodynamics problem is this:

Begin by carefully reading the problem statement completely through.

Step 1. Make a sketch of the system and put a dashed line around the system boundary.

- Step 2. Identify the unknown(s) and write them on your system sketch.
- Step 3. Identify the type of system (closed or open) you have.
- Step 4. Identify the process that connects the states or stations.
- Step 5. Write down the basic thermodynamic equations and any useful auxiliary equations.
- Step 6. Algebraically solve for the unknown(s).
- Step 7. Calculate the value(s) of the unknown(s).
- Step 8. Check all algebra, calculations, and units.

The process is this:

```
Sketch \rightarrow Unknowns \rightarrow System \rightarrow Process \rightarrow Equations \rightarrow Solve \rightarrow Calculate \rightarrow Check
```

For example, if you have a closed system that is initially at 14.7 psia with a specific volume of  $0.500 \text{ ft}^3/\text{lbm}$  and by some process it ends up at 200. psia at a quality of 90.0%, you should write this information on your work sheet in the following form (always be sure to include the units on these values):

$\underline{State 1} \xrightarrow{Process path}$	State 2
$p_1 = 14.7  \text{psia}$	$p_2 = 200.  \text{psia}$
$v_1 = 0.500  \text{ft}^3/\text{lbm}$	$x_2 = 0.900$

For example, if you have a flow stream entering the system at station 1 with a temperature of 300.°C and a pressure of 1.00 MPa, and a flow stream exiting the system at station 2 with a specific volume of 26.3  $\text{m}^3/\text{kg}$  and a quality of 99.0%, you should write this information on your work sheet as (always be sure to include the *units* on these values):

$$\underbrace{\text{Station 1}}_{p_1 = 1.00 \text{ MPa}} \xrightarrow{\text{Process path}} \underbrace{\text{Station 2}}_{\nu_2 = 26.3 \text{ m}^3/\text{kg}}$$

$$\underbrace{r_1 = 300.^\circ\text{C}}_{x_2 = 0.990}$$

Here, too, we are trying to identify two independent property values at each station, because in simple systems, they fix the state of the material at that station.

Notice that, for "simple" thermodynamic systems, we always are looking for the values of two independent properties in each state. These two property values fix (i.e., determine) the state and we can then find the values of any of the other properties needed at that state.

Often a problem statement gives only one property value at a system state. In this case, the remaining independent property value at that state is usually given by the process path statement that indicates how that state was achieved (e.g., an isothermal process tells us that  $T_2 = T_1$ ) or else it may be a problem unknown to be determined.

If it is an open system, we are interested in any changes that occur in the system bulk properties of the system plus all the properties of the entering and exiting flow streams. Flow stream properties are referred to as monitoring *station* properties, to clearly separate them from bulk system properties.

Step 4. Now **identify the process connecting the state or stations**. The process path statement is usually given in technical terms such as *a closed, rigid vessel*, meaning an isochoric (or constant volume) process will occur. Proper identification of the process path is very important, because it often provides numerical values for state properties (e.g.,  $v_2 = v_1$  for a closed, rigid vessel) or heat, work, or other thermodynamic quantities (e.g., an insulated or adiabatic system has  ${}_1Q_2 = \dot{Q} = 0$ , an aergonic system has  ${}_1W_2 = \dot{W} = 0$ , and so forth). When two independent property values are given in the problem statement for each state or station of the system, the process path is not necessary unless it provides values for heat, work, kinetic energy, or potential energy.

Step 5. Write down all the basic equations. Your work sheet should now have all the details of the problem on it and you should not have to look at the problem statement again. The actual solution to the problem is begun by automatically writing down (whether you think you need them or not) all the relevant basic equations. Thermodynamics has only three basic equations:

- a. The conservation of mass (which is also called the mass balance).
- **b.** The first law of thermodynamics (which is also called the energy balance or the conservation of energy).
- c. The second law of thermodynamics (which is also called the entropy balance).

In closed systems, the conservation of mass is automatically satisfied and need not be written down. Also, since the entropy balance is not be introduced until Chapter 7, it does not enter into the solution of any problems until then. So, for solving the closed system problems of Chapter 5, there is really only one relevant basic equation: the first law of thermodynamics. In solving the open system problems of Chapter 6, there are two relevant basic equations: the conservation of mass and the first law of thermodynamics.

Write any necessary auxiliary equations. All the equations developed in this book that are not one of the three basic equations discussed previously are called *auxiliary equations*. For example, all equations of state (ideal gas and incompressible materials), all work mode equations (mechanical, electrical, etc.), all heat mode equations (conduction, convection, radiation), all property-defining equations (specific heats,

The easiest way to show the process path on your work sheet is to write the statement "Process: process name" on a connecting arrow between the state or station data sets. In the closed system example used in step 3, if the state change occurs in a closed, rigid vessel and we do not know the final quality, then we would write

<u>State 1</u> $\xrightarrow{\text{Process: } v = \text{ constant}}$	State 2
$p_1 = 14.7  \text{psia}$	$p_2 = 200.  \text{psia}$
$v_1 = 0.500  \text{ft}^3 / \text{lbm}$	$v_2 = v_1 = 0.500 \mathrm{ft}^3/\mathrm{lbm}$

And, if the open system of step 3 is operated at a constant pressure (i.e., an isobaric process) and we do not know the final quality, then we would write

Process: $p = constant$	
Station 1	Station 2
$p_1 = 1.00 \text{ MPa}$	$v_2 = 26.3 \mathrm{m}^3/\mathrm{kg}$
$T_1 = 300.^{\circ}C$	$p_2 = p_1 = 1.00 \text{ MPa}$

Always write down the complete *general* form of the basic equations. Do not try to second-guess the problem by writing the shorter specialized forms of the basic equations that were developed for specific applications. Then, cross out all terms that vanish as a result of given constraints or process statements. For example, for a closed, adiabatic, stationary system, we write the energy balance as (see Eq. (4.20), where we have used the abbreviation  $KE = mV^2/2g_c$  and  $PE = mgZ/g_c$ )

$$1Q_2 - {}_1W_2 = m(u_2 - u_1) + KE_2 - KE_1 + PE_2 - PE_1$$
  
= 0 (adiabatic (insulated) system) = 0 (stationary - i.e., not moving)

Notice that we write why each crossed out term vanishes ("adiabatic" and "stationary" in this case). This makes the solution easier to follow and to check later if the correct answer was not obtained.

Unlike in some other engineering subjects, you will not be able to find all the algebraic manipulations already done for you in example problems within the text or by the instructor in class. There are simply too many possible variations on a problem theme to do this. Therefore, you have to carry out the mathematical manipulations suggested here to develop your own working formulae in almost every problem. This is a fact of thermodynamic problem solving.

By this point you should be able to see your way to the end of the problem, because the mechanism for finding each of the unknowns should now be clear. Determine the units on each value calculated and make sure that all values that are added together or subtracted from each other have the same units. Often one of the unknowns is needed to find another; for example, you may need to find  $_1W_2$  from a work mode auxiliary equation to solve for  $_1Q_2$  from the energy balance equation.

enthalpy, etc.), and all specialized equations (such as  $KE = mV^2/2g_c$ , etc.) are auxiliary equations. If the problem statement describes a mechanical, electrical, or other work mode, then write the equation for calculating the value of that work mode. Auxiliary equations ultimately provide numerical values for use in the basic mass, energy, and entropy balance equations.

Step 6. Algebraically solve for the unknown(s). Do not calculate anything yet. By algebraically manipulating the basic and auxiliary equations you should be able to develop a separate equation for each unknown. Remember, you can solve for only as many unknowns as you have independent equations. All of the basic equations and most of the auxiliary equations are independent, so many times unknowns are determined directly from an auxiliary equation. For example, in the problem statements dealing with closed systems, we have only *one* applicable basic equation, the first law of thermodynamics (the energy balance). Therefore, if there is more than one unknown in these problem statements, then all but one of these unknowns must be determined directly from an appropriate auxiliary equation.

Step 7. **Calculate the value(s) of the unknown(s)**. Once all the algebra has been completed, then and only then should you begin to calculate numerical values.

Step 8. Check all algebra, calculations, and units. This is self-explanatory, but pay particular attention to checking the units. With the calculational accuracy of today's inexpensive electronic calculators and microcomputers, most of your errors occur as a result of poor units handling rather than from numerical manipulations.

These eight steps are illustrated in detail in the examples in the next chapter. They will lead you through even the most difficult thermodynamic problems. Once you become familiar with them, the solutions flow quite rapidly and naturally. It must be emphasized that these steps are not the only solution technique possible, but they have proven successful for many engineering students.

# 4.15 HOW TO WRITE A THERMODYNAMICS PROBLEM

A good test of your problem solving skills is to see whether or not you can write a thermodynamics problem that can be solved. The technique of writing your own thermodynamics problem is just the reverse of solving one. It is as simple as A, B, C.

- **A.** First, you first decide (1) the type of system (closed or open) you want to use, (2) the equations you want to use in the solution (thermodynamic laws, equations of state, work mode equation, and so forth), and (3) the unknown(s) you want to find in the solution.
- **B.** Next, you write a short story that provides physical motivation for the problem that contains all the numerical values necessary to find state properties and any geometry, height, or velocity information needed to solve for your chosen unknowns.
- **C.** Finally, you solve your problem in a forward direction to see if you have specified all the necessary information for someone to produce an accurate solution.

This is easier than it sounds. First, let us look at the equations that can be used in a problem solution.

### A. Select the working equations and unknowns

The problem unknowns can be any of the variables carried within the basic laws of thermodynamics and any of the related auxiliary equations introduced thus far. For simplicity, let us limit the discussion to a closed system analysis. The general closed system energy balance is

$${}_{1}Q_{2} - {}_{1}W_{2} = m \left[ (u_{2} - u_{1}) + \frac{V_{2}^{2} - V_{1}^{2}}{2g_{c}} + \frac{g(Z_{2} - Z_{1})}{g_{c}} \right]_{\text{system}}$$

and the general closed system energy rate balance is

$$\dot{Q} - \dot{W} = \frac{d}{dt} \left( mu + \frac{mV^2}{2g_c} + \frac{mgZ}{g_c} \right)_{\text{system}}$$

Any of the variables listed in these equations can be an unknown in a problem statement. In addition to the basic balance equations, we have numerous auxiliary equations, such as

- Equations of state for ideal gases, incompressible fluids, or other materials.
- Process path equations such as polytropic, isobaric, and the like.
- Various work mode equations for mechanical, electrical, and other work modes.

List all the *basic* (thermodynamics laws) and *auxiliary* equations you want the person who solves your problem to use in the solution. Then choose the variables you want to use as unknowns. Remember, you need as many independent equations in your list of equations as the number of unknowns you choose, so do not choose too many. Then, assign numerical values to all the remaining variables in the equations that are to be used to solve for the unknowns. Do not be too concerned about the actual values you pick at this point; if you choose the wrong values, it will show up in step C, and you can correct them later.

# B. Write a short story that contains all the information needed to solve the problem

It would be helpful if we could categorize to some degree the wide variety of problem types or scenarios commonly encountered in thermodynamics. The first classification is by the *thermodynamic process* used in the problem scenario, the second classification is by the *engineering technology* used in the problem scenario, and the third is by *problem unknown*. Since the number of variations within these classifications is quite large, they are explained in detailed here.

**Problem classification by thermodynamic process**. A problem statement could involve more than one process or involve unknown processes. Therefore, the process for changing the state of a system could be the focal point of a problem statement. For example, we might want to find how the temperature changes during a constant pressure process. This would then be the central theme of the problem statement.

**Problem classification by problem technology**. The list of possible engineering technologies is much longer than the list of known processes. Actually, any device or technology can be analyzed thermodynamically. A series of "typical" technology based scenarios appear in engineering thermodynamics textbooks. For example, you might want to find the work required to compress a gas with a piston, the change in temperature across a nozzle, the power produced by a turbine, and so forth. Then, the problem statement focuses on these technologies, providing numerical values for all the variables except the problem unknowns.

**Problem classification by problem unknown**. These problems are usually the simplest, since they do not depend on a specific technology or process path. The unknowns are simply calculated directly from the thermodynamic laws (i.e., *Q*, *W*, KE, PE, etc.) or from an auxiliary equation (i.e., the ideal gas equation, etc.).

## C. Solve the problem in the forward direction

Here you (or a friend) must actually solve the problem you wrote using the data you provided in the problem statement. You will usually find that you get stuck part way through the problem and have to go back and modify the problem statement. That is OK, do it quickly and go on with the solution. Sometimes, values you originally chose cannot be found easily in the tables or are unreasonable (for example, maybe you wanted a state to be a vapor but the values you originally specified for pressure and temperature are for a liquid). Using the tables in the tables book and your emerging solution, change the original values in your problem statement so that the problem solution moves along smoothly. Be careful to check the units on each calculation.

# Now Let us Write a Thermodynamics Problem

Step A. We limit it to a closed system and use the energy balance as our primary equation:

$$_{1}Q_{2} - _{1}W_{2} = m \left[ (u_{2} - u_{1}) + \frac{V_{2}^{2} - V_{1}^{2}}{2g_{c}} + \frac{g(Z_{2} - Z_{1})}{g_{c}} \right]_{\text{system}}$$

**Choose the material**. Let the system contain an ideal gas. Then auxiliary equations pv = RT and  $u_2 - u_1 = c_v(T_2 - T_1)$  can be used.

**Choose the unknowns**. With two independent equations, we can have two unknowns. Let us choose  ${}_{1}Q_{2}$  and  $p_{2}$  as the unknowns. We can solve for  ${}_{1}Q_{2}$  from the energy balance and solve for  $p_{2} = RT_{2}/v_{2}$ . If we put the system in a rigid container, then  ${}_{1}W_{2} = m \int p dv = 0$ , because for a sealed rigid container, v = constant, then dv = 0. Let us also add the condition that the process must be isothermal, then  $T_{2} = T_{1}$  and thus  $u_{2} - u_{1} = c_{v}(T_{2} - T_{1}) = 0$ . Further, let us also require that  $V_{2} = V_{1}$ , then the energy balance reduces to

$$_{1}Q_{2} - 0 = m \left[ 0 + 0 + \frac{g(Z_{2} - Z_{1})}{g_{c}} \right]_{\text{syster}}$$

Now all we need to do is specify m,  $Z_1$ , and  $Z_2$  and we can compute  ${}_1Q_2$ .

Step B. The next step is to write a scenario, or a short story, that uses these processes and values to create a thermodynamic problem. Let us try this:

There are 5.00 kg of hydrogen gas (an ideal gas) at 20.0°C and 0.300 MPa sealed inside a wooden barrel (a rigid container) at the top of Niagara Falls. The barrel is not insulated and is maintained at a constant temperature (i.e., isothermal) as it travels over the falls in contact with the water. Determine

a. The heat transfer from the barrel as it travels 50.0 m vertically between the top and bottom of the falls.b. The final pressure inside the barrel at the bottom of the falls.

Note that the problem scenario does not have to be deadly serious, you can write problem statements around anything your imagination can conceive.

Step C. Now we must work the problem in the forward direction to see if all the necessary information has been provided, so let us try it.

#### Solution

The problem solving technique requires that we start by reading the problem statement carefully.

Step 1 ask us to draw a sketch of the system (the barrel going over the Niagara Falls, see Figure 4.22) and identify the material in the system, it is the hydrogen in the barrel.

Step 2 asks us to identify the unknowns. Even though we just wrote the problem statement, it is important to read it again, *carefully*, to check for errors and completeness. The problem statement should contain clarifying statements so that the reader need not make any unreasonable assumptions. For example, in our problem statement, we identified the hydrogen as an *ideal gas*, because it is not obvious to a beginning thermodynamics stu-

dent which materials behave like an ideal gas and which do not. Also, while it may be obvious to you when you wrote the problem statement that a barrel is to be modeled as a *sealed*, *rigid container*, it is advisable to tell the reader this in clear terms, since the purpose of the problem should be to test the problem solving skills of the reader, not his or her ability to read your mind about how to interpret unfamiliar things. The unknowns here are clearly specified in items (a) and (b) at the end of the problem statement. They are find  ${}_{1}Q_{2}$  and  $p_{2}$ .

*Step 3* asks us to identify the system's type and its states. The system here is closed (because the barrel is *sealed*). We should be able to identify the system states from the information given in the problem statement:

State 1
 State 2

 
$$T_1 = 20.0^{\circ}$$
C
  $T_2 = T_1 = 20.0^{\circ}$ C

  $p_1 = 0.300$  Mpa
 ?

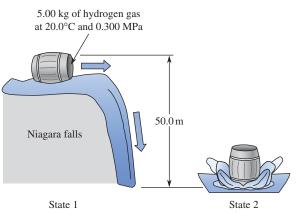


FIGURE 4.22 A barrel going over Niagara Falls.

We have now identified two properties in the first state but only one in the second. This is a common structure at this point in the solution. The missing property must come from somewhere else in the solution, either from the process path or from the working equations used in the solution.

In *step 4* we have to identify the process path taken by the system as it moves between states 1 and 2. Note that, since the system is at a *constant temperature* and is a *sealed, rigid container*, the process path here has both constant temperature and constant volume and mass, so it is also has a constant specific volume. Now, we can add the process path line to the state information and the missing second state property, so that it looks like this:

State 1	T = costant and $v = constant >$	State 2
$T_1 = 20.0^{\circ} \text{C}$		$T_2 = T_1 = 20.0^{\circ}$ C
$p_1 = 0.300 \mathrm{Mpa}$		$v_2 = v_1 = RT_1/p_1$

We now have two properties in each state and can continue with the solution. Note that we do not need to calculate the value of  $v_2$  yet, since we are not sure we need it in the solution.

*Step 5* is to write the basic equations. Since this is a closed system, the conservation of mass equation yields no useful information, as the mass of the system is constant. However, the conservation of energy (the *first law of thermodynamics*) is very useful here:

$$_{1}Q_{2} - {}_{1}W_{2} = m \left[ (u_{2} - u_{1}) + \frac{V_{2}^{2} - V_{1}^{2}}{2g_{c}} + \frac{g(Z_{2} - Z_{1})}{g_{c}} \right]_{\text{system}}$$

Next, we write all the relevant auxiliary equations. If you do not know whether an auxiliary equation is relevant or not, write it down anyway and decide later. Let us start with equations of state. If the material in the system were steam or refrigerant or anything for which there is a table in the tables book, we would use those tables rather than an equation of state. However, hydrogen was given in the problem statement as an *ideal gas*, so we can write its equations of state as

$$pv = RT$$
 and  $u_2 - u_1 = c_v(T_2 - T_1)$ 

Note that the first equation of state can be used as both  $p_1v_1 = RT_1$  and  $p_2v_2 = RT_2$ .

Next, let us look at work mode equations. No rotating shafts or wires cross the system boundary nor has any reference been made in the problem statement to any electric or magnetic fields. Consequently, no shaft, electrical, polarization, magnetic, or other work mode is present. We also need to check for moving boundary work,  $({}_{1}W_{2})_{\text{moving boundary}} = m/pdv$ . Since the system is *closed* (the mass is constant) and *rigid* (so the volume is constant), the specific volume (total volume divided by mass) is constant. Then dv = 0 and there is no moving boundary or any other type of work. So,  ${}_{1}W_{2} = 0$ .

At this point we should also identify any changes in kinetic or potential energy. Our problem statement specifies the change in height over the falls as 50.0 m, but it does not mention anything about velocity. The intent here is to have the initial and final velocities of the system be the same, but that might be too much to ask the reader to assume. Therefore, we should alter the problem statement by replacing the word *Determine* with the phrase *Assuming the initial and final velocities of the barrel are the same, determine*. Then, the problem statement reads as follows:

Five kilograms of hydrogen gas (an ideal gas) at 20.0°C and 0.300 MPa are sealed inside a wooden barrel (a sealed, rigid container) at the top of Niagara Falls. The barrel is not insulated and is maintained at a constant temperature (i.e., isothermal) as it travels over the falls in contact with the water. Assuming the initial and final velocities of the barrel are the same, determine

- a. The heat transfer from the barrel as is travels 50.0 m vertically between the top and bottom of the falls.
- **b.** The final pressure inside the barrel at the bottom of the falls.

In *step 6* we are ready to algebraically solve for the unknowns. From the energy balance, we can solve for the heat transfer required in part (a) as

$$_{1}Q_{2} = m \left[ (u_{2} - u_{1}) + \frac{V_{2}^{2} - V_{1}^{2}}{2g_{c}} + \frac{g(Z_{2} - Z_{1})}{g_{c}} \right]_{\text{system}} + {}_{1}W_{2}$$

Now, we incorporate our earlier results that  $_1W_2 = 0$  and  $u_2 - u_1 = c_v(T_2 - T_1) = 0$ , because  $T_2 = T_1$  here (the process is also isothermal). Our latest rendition of the problem statement makes it clear that  $V_2 = V_1$ , and when these conditions are incorporated into the energy balance, we obtain our final equation for the heat transfer as

$$_{1}Q_{2} = m \left[ 0 + 0 + \frac{g(Z_{2} - Z_{1})}{g_{c}} \right]_{\text{system}} + 0 = \frac{mg}{g_{c}} (Z_{2} - Z_{1})$$

and, from the equation of state, we can determine the solution to part (b) as

$$p_2 = RT_2/v_2 = RT_1/v_2 = RT_1/v_1 = p_1$$

since  $T_2 = T_1$  and  $v_2 = v_1$ .

Step 7 allows us to calculate the values of the unknowns:

(a) 
$$_{1}Q_{2} = \frac{(5.00 \text{ kg})(9.81 \text{ m/s}^{2})}{1}(0-50.0 \text{ m}) = -2450 \text{ kg} \cdot \text{m}^{2}/\text{s}^{2}$$
  
= -2450 N \cdot m = -2450 J = -2.45 kJ

and

(b)  $p_2 = p_1 = 0.300 \text{ MPa}$ 

The negative sign in the answer for part (a) tells us that the heat transfer is *out of* the system. Note that the answer in part (b) was not the result of a complex calculation. However, it did result from a rather complex analysis and, therefore, is not trivial. Also note that we did not need the value of  $v_2$  in the solution of the problem, so it would have been a waste of time to have calculated it early in the solution.

In *step 8*, since the solution now seems to work well, the problem statement is complete and accurate. We should now check all the algebra, units, and calculations before creating and solving additional problems with similar or different scenarios.

#### Exercises for the problem solved in Steps 1–8

- 1. Rewrite this problem and make the barrel insulated but not isothermal. (Can it be both insulated and isothermal?) Resolve the problem with these new conditions. Is any additional information needed to find  $T_2$  and  $p_2$ ?
- **2.** Write a thermodynamics problem about a computer chip. Look up the steady state voltage and current required by a typical computer chip in a handbook and supply these values in the problem statement. This is a closed system, and the chip cannot be insulated (otherwise, it would overheat). Use the energy rate balance in the formulation of your problem scenario.
- **3.** Write a thermodynamics problem about an electrical generator. Use the closed system energy rate balance. Make the process steady state. You may have the generator insulated or uninsulated. Note that there are two work modes here, shaft work and electrical work.
- **4.** Write a thermodynamics problem about an airplane. Make it a closed system and have it change altitude and speed. Choose an appropriate unknown and provide all the necessary values for the remaining variables.

## **SUMMARY**

In this chapter, we discover that the first law of thermodynamics is simply the conservation of energy principle. Since energy is conserved in all actions, the change in a system's energy can be equated to the net transport of energy into the system. Only three possible energy transport mechanisms are available to us: (1) heat transport of energy (commonly called *heat transfer*), (2) work transport of energy (commonly called *work*), and (3) energy transported with a mass flowing across a system's boundary. This information produced the very powerful energy balance and energy rate balance equations.

The general closed system energy balance:

$${}_{1}Q_{2} - {}_{1}W_{2} = (E_{2} - E_{1})_{\text{system}}$$
  
=  $m[(u_{2} - u_{1}) + (V_{2}^{2} - V_{1}^{2})/(2g_{c}) + (Z_{2} - Z_{1})g/g_{c}]_{\text{system}}$ 

The general closed system energy rate balance:

$$\dot{Q} - \dot{W} = (dE/dt)_{\text{system}} = (m\dot{u} + mV\dot{V}/g_c + mg\dot{Z}/g_c)_{\text{system}}$$

The general open system energy rate balance:

$$\dot{Q} - \dot{W} + \sum \dot{E}_{\max_{\text{flow}}} = (d/dt) (mu + mV^2/2g_c + mZg/g_c)_{\text{system}}$$

Work modes of energy transport are not discussed in any course outside of thermodynamics and are very important for utilizing the full capacity of the first law of thermodynamics. We need to understand and master the work mode auxiliary equations, because they are often required in the solution of thermodynamic problems. Some of the important work mode auxiliary equations are given in Table 4.10. The associated power equations are given in Table 4.6 of the text.

The local equilibrium postulate allows us to deal with nonequilibrium states, and the state postulate defines the number of independent thermodynamic properties required to determine the local equilibrium state (two, for a simple system).

Heat transport of energy (heat transfer  ${}_{1}Q_{2}$  and heat transfer rate  $\dot{Q}$ ) is categorized into three modes: (1) conduction, (2) convection, and (3) radiation. Heat transfer is sufficiently important to mechanical engineers that most curricula have separate heat transfer courses. Consequently, the details of this subject are not emphasized in a thermodynamics course. The heat transfer rate modes are summarized in Table 4.11.

Generally, if you are asked to determine a heat transfer in a problem statement, you should calculate it from the first law energy balance rather than from one of the heat transfer mode auxiliary equations.

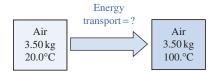
Table 4.10 Work Mode Auxiliary Equations				
Work Mode	Equation			
Moving boundary (general)	$\binom{1}{1}W_2_{\text{boundary}} = \int_1^2 \rho  d \forall$			
Polytropic moving boundary ( $n \neq 1$ )	$\binom{1}{1}W_2$ polytropic $(n \neq 1) = \frac{mR}{1-n}(T_2 - T_1)$ ideal gas moving boundary			
Rotating shaft	$\binom{1}{1}W_2_{\text{rotating}} = \int_1^2 \vec{\tau} \cdot d$			
Elastic	$(_1W_2)_{\text{elastic}} = -\int_1^2 \sigma \forall d\epsilon$			
Surface tension	$\binom{1}{1}W_2_{\text{surface}} = -\int_1^2 \sigma_s dA$			
Electrical current	$\binom{1}{1}W_2_{\text{electrical}} = \int_1^2 \phi i  dt$			
Electrical polarization	$\binom{1}{1}W_2_{\text{electrical}} = -\int_1^2 E  dP$			
Magnetic	$({}_{1}W_{2})_{magnetic} = -\mu_{0} \forall (1 + \chi_{m}) \left(\frac{H_{2}^{2} - H_{1}^{2}}{2}\right)$			
Chemical	$\binom{1}{W_2}_{\substack{\text{chemical}\\ \mu_i = \text{ constant}}} = -\sum_{i=1}^k \mu_i (m_2 - m_1)_i$			
Mechanochemical	$(_1W_2)_{\text{mechanochemical}} = \int_1^2 f  d\ell$			

Table 4.11         Heat Transfer Rate Modes					
Heat Transfer Mode	Equation				
Conduction	$\dot{Q}_{cond} = -k_t A \left( \frac{dT}{dx} \right)$				
Convection	$\dot{Q}_{conv} = hA(T_{\infty} - T_{s})$				
Radiation	$\dot{Q}_{\rm rad} = F_{1-2}\varepsilon_1 A_1 \sigma (T_2^4 - T_1^4)$				

Finally, we study a special technique that maps the solution of any thermodynamic problem. If you follow the format given in Figure 4.21, you will breeze through the solution maze. But you must follow it religiously and take no shortcuts. As an extension of your problem solving skills, you are also shown how to write and solve your own thermodynamic problems. If you can do this successfully, you have mastered the subject.

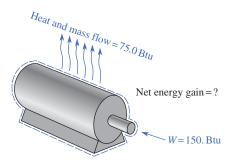
# Problems (\* indicates problems in SI units)

1.\* Determine the energy transport required to increase the temperature of 3.50 kg of air from 20.0 to 100.°C (Figure 4.23). Assume the air is stationary and behaves as an ideal gas with constant specific heats.



#### FIGURE 4.23

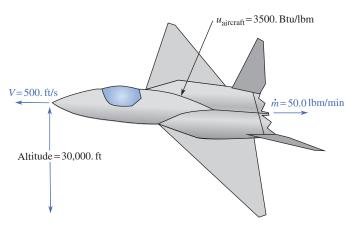
- 2.\* Determine the energy transport necessary to decrease the temperature of 15.0 kg of methane from 500. to 20.0°C. Assume the methane is stationary and behaves as an ideal gas with constant specific heats.
- Determine the gain in energy of a stationary system of 5.00 lbm of argon whose temperature is increased from 70.0 to 1000.°F. Assume ideal gas behavior with constant specific heats.
- **4.**\* Determine the gain in energy of a stationary system of 11.0 kg of oxygen whose pressure is increased from 0.100 to 100. MPa isothermally. Assume ideal gas behavior with constant specific heats.
- **5.** If 150. Btu are transported into a system via a work mode while 75.0 Btu are removed via heat transfer and mass flow modes (Figure 4.24), determine the net energy gain for this system.



# FIGURE 4.24

Problem 5.

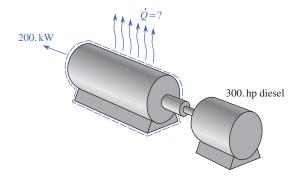
**6.** A jet aircraft with a constant specific internal energy of 3500. Btu/lbm consumes fuel at a rate of 50.0 lbm/min while flying horizontally at an altitude of 30,000. ft with a constant velocity of 500. ft/s (Figure 4.25). Determine the net energy transport rate of the aircraft.



#### FIGURE 4.25

Problem 6.

- **7.** An automobile transmission has 175 hp of power entering from the engine, 167 hp leaving to the wheels, while losing 5000. Btu/h to the surroundings as heat. What is the net energy transport rate of the transmission?
- 8. To keep the transmission in the previous problem from overheating, it was decided to cool it by circulating a coolant through its case. If the coolant enters the transmission with a mass flow energy rate of 10.0 Btu/s, what is its mass flow energy rate as it leaves the transmission?
- **9.** Determine the heat transfer rate, in Btu/h, required to cool a 200. kW electric generator that is driven by a 300. hp diesel engine (Figure 4.26). Note: The generator runs cool if it has a zero net energy transport rate.



#### FIGURE 4.26

Problem 9.

10. In a stationary dynamometer test, an internal combustion automobile engine has a fuel energy input rate of 1.90 million Btu/h while producing 150. hp of output power. What other energy transport mechanisms are present and what are their magnitudes. Assume that the net energy transport rate is zero.

Problem 1.

- **11.\*** Determine the heat transfer per kg necessary to raise the temperature of a closed rigid tank of saturated water vapor originality at 0.140 MPa to a temperature of 800.°C.
- **12.** A closed rigid vessel of volume 5.00 ft<sup>3</sup> contains steam at 100. psia with 83.91% moisture. If 9490.4 Btu of heat are added to the steam, find the final pressure and quality (if wet) or temperature (if superheated).
- **13.\*** A closed rigid vessel having a volume of 0.566 m<sup>3</sup> is filled with steam at 0.800 MPa and 250.°C. Heat is transferred from the steam until it exists as saturated vapor. Calculate the amount of heat transferred during this process.
- 14. A sealed, rigid tank of 10.0 ft<sup>3</sup> capacity is initially filled with steam at 100. psia and 500.°F. The tank and its contents are then cooled to 260.°F. Find (a) the final quality in the container and the amounts of liquid water and water vapor (in lbm), and (b) the amount of heat transfer required (in Btu).
- **15.\*** A sealed rigid vessel contains 5.00 kg of water (liquid plus vapor) at 100.°C and a quality of 30.375%.
  - a. What is the specific volume of the water?
  - **b.** What is the mass of water in the vapor phase?
  - **c.** What would be the saturation pressure and temperature of this water if it had the specific volume determined in part a and a quality of 100%?
  - **d**. What heat transfer would be required to completely condense the saturated vapor of part c into a saturated liquid?
- 16. One pound of saturated liquid water at a pressure of 40.0 psia is contained in a rigid, closed, stationary tank. A paddle wheel does 3000. ft·lbf of the work on the system, while heat is transferred to or from the system. The final pressure of the system is 20.0 psia. Calculate the amount of heat transferred and indicate its direction.
- **17.** Identify the following as either point or path functions: **a.**  $u^2 + 3u - 5$ .

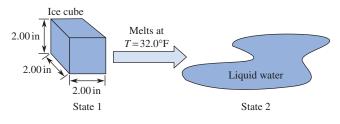
**b.** 
$$T(h^2 - u^2) - 3(u - pv) + 4$$
.

$$\sin u^3 + \sin h^3$$
.

- **d.**  $\int_{1}^{2} \Psi dp$ , where  $\Psi = \Psi(p)$ .
- Identify the following as either point or path functions:
   a. *RT*/*v*.
  - **b.**  $\int_{1}^{2} p \, d\Psi$ , where  $p = p(\Psi)$ . **c.** h + pv. **d.**  $u + V^2/2g_c + gZ/g_c$ .
- **19.** Explain whether  $u_2 u_1 = \int_1^2 c_v dT$  is a point or a path function for a given system.
- **20.** Explain whether  $h_2 h_1 = \int_1^2 c_p dT$  is a point or a path function

for a given system.

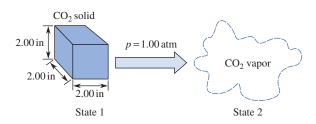
- **21.** Explain the meaning of the notation  ${}_1Q_2$  and  ${}_1W_2$ . Why do we not write  ${}_1E_2$ ,  ${}_1u_2$ , or  ${}_1h_2$ ?
- 22.\* Determine the moving boundary work transport of energy when4.5 kg of water expands at constant pressure from saturatedliquid to saturated vapor while at 20.0°C.
- **23.** Determine the moving boundary work done by the atmosphere (14.7 psia) as a cube of ice 2.00 in on a side melts into a pool of liquid water (Figure 4.27). At 32.0°F, the density of ice is 57.2 lbm/ft<sup>3</sup> and that of liquid water is 62.4 lbm/ft<sup>3</sup>.



### FIGURE 4.27

Problem 23.

24. Determine the moving boundary work done by a cube of solid CO<sub>2</sub> 2.00 in on a side as it vaporizes at atmospheric pressure (14.7 psia) (Figure 4.28). The density of solid CO<sub>2</sub> is 97.561 lbm/ft<sup>3</sup> and that of CO<sub>2</sub> vapor is 0.174 lbm/ft<sup>3</sup>.



## FIGURE 4.28

Problem 24.

- **25.** A weather balloon is filled with helium at 50.0°F so that its volume is 500. ft<sup>3</sup>. The balloon is left anchored in the sun and its temperature rises to 110.°F. How much moving boundary work is done by the balloon on the atmosphere as its volume increases due to the increase in temperature? Assume that helium is an ideal gas and the balloon skin is sufficiently thin that the pressure in the balloon remains approximately atmospheric.
- 26.\* Suppose 2.00 m<sup>3</sup> of air (considered an ideal gas) is initially at a pressure of 101.3 kPa and a temperature of 20.0°C. The air is compressed at a constant temperature in a closed system to a pressure of 0.500 MPa (Figure 4.29). (a) How much work is done on the air to compress it? (b) How much energy is transferred as heat during the compression process?

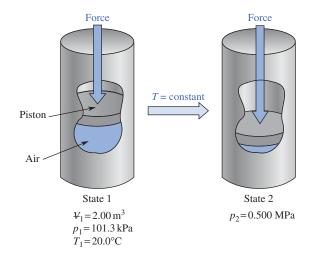
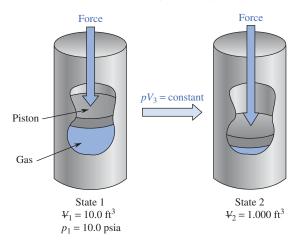


FIGURE 4.29

Problem 26.

- **27.** Show that the first law of thermodynamics requires that, for an ideal gas with a constant specific heat ratio  $c_p/c_v = k$  undergoing a polytropic process (i.e.,  $pv^n$  = constant),
  - **a.** *n* must be greater than *k* for  $T_2 < T_1$  when there is heat transfer from the gas.
  - **b.** *n* must be less than *k* for  $T_2 < T_1$  when there is a heat transfer to the gas.
- **28.** Find the moving boundary work done on a gas in compressing it from  $\Psi_1 = 10.0 \text{ ft}^3$ ,  $p_1 = 10.0 \text{ psia to } \Psi_2 = 1.000 \text{ ft}^3$  according to the relation  $p \Psi^3 = \text{constant}$  (Figure 4.30).



#### **FIGURE 4.30**

Problem 28.

- **29.\*** A brilliant young engineer claims to have invented an engine that runs on the following thermodynamic cycle:
  - **a.** An isochoric pressurization from  $p_1$  to  $p_2 = p_1$ .
  - **b.** An isobaric expansion from  $\Psi_2$  to  $\Psi_3 = 2\Psi_2$ .
  - **c.** An isochoric depressurization from  $p_3$  to  $p_4 = p_1$ .
  - **d.** An isobaric compression back to the initial state,  $p_1$ ,  $\Psi_1$ . Determine the net moving boundary work done during this cycle if  $p_1 = 25.0$  kPa and  $\Psi_1 = 0.0300$  m<sup>3</sup>. Sketch this cycle on a  $p - \Psi$  diagram.
- **30.\*** A balloon filled with air at 0.100 MPa-absolute is heated in sunlight. As the balloon is heated, it expands according to the following pressure-volume relation:

 $p = 0.1 + 0.15 \forall + 0.06 \forall^2$ 

where *p* is in MPa and  $\forall$  is in m<sup>3</sup> (Figure 4.31). Determine the moving boundary work transport of energy as the balloon expands from 1.00 to 2.00 m<sup>3</sup>.

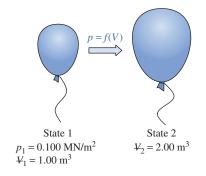


FIGURE 4.31 Problem 30. 31. One lbm of an ideal gas with molecular weight 6.44 lbm/ lbmole is compressed in a closed system from 100. psia, 600. R to a final specific volume of 8.00 ft<sup>3</sup>/lbm. At all points during the compression, the pressure and specific volume are related by

$$p = 50 + 4v + 0.1v^2$$

where *p* is in psia and *v* is in  $ft^3/lbm$ . Determine the moving boundary work required and the heat transfer during this compression if the gas has a constant volume specific heat of 0.200 Btu/(lbm·R).

- **32.** Three lbm of a substance is made to undergo a reversible expansion process within a piston-cylinder device, starting from an initial pressure of 100 psia and an initial volume of 2.00 ft<sup>3</sup>. The final volume is 4.00 ft<sup>3</sup>. Determine the moving boundary work produced by this expansion for each of the following process paths. Note which process produces the maximum work and which produces the minimum.
  - **a.** Pressure remains constant (p = K)
  - **b.** Pressure times volume remains constant  $(p \forall = K)$ .
  - **c.** Pressure is proportional to volume  $(p = K \forall )$ .
  - **d.** Pressure is proportional to the square of volume  $(p = K \Psi^2)$ .
  - e. Pressure is proportional to the square root of volume
    - $(p = K_{\sqrt{V}})$ , where *K* is a constant in each case.
- **33.**\* The magnitude of the torque *T* on a shaft is given in  $N \cdot m$  by

#### $T = 6.3 \cos \theta$

where  $\theta$  is the angular displacement. If the torque and displacement vectors are parallel, determine the work required to rotate the shaft through one complete revolution.

**34.** The magnitude of the torque vector normal to the axis of a shaft is given in ft lbf by

$T = 21.7 \sin \theta$	for $0 < \theta \le \pi$
= 0	for $\pi < \theta \leq 3\pi/2$
= 50.4	for $3\pi/2 < \theta \le 2\pi$

Determine the work done in one complete revolution of the shaft.

**35.** When the torque and angular displacement vectors are parallel, the torque displacement relation for the drive shaft of a 1909 American Underslung automobile is given by

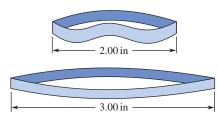
#### $T\theta^n = K$

where *K* and *n* are constants. Determine a general formula for the shaft work when (a) n = 1.0, and (b)  $n \neq 1.0$ .

- **36.** How much elastic work is done in uniaxially stretching an initially unstrained elastic steel bar (Young's modulus =  $3.0 \times 10^7$  psi = constant) whose volume (also a constant) is 5.00 in<sup>3</sup> to a total strain of 0.00200 in/in?
- **37.** When a rubber band is stretched, it exerts a restoring force (*F*) that is a function of its initial length (*L*) and displacement (*x*). For a certain rubber band this relation is

$$F = K \left[ \frac{x}{L} + \left( \frac{x}{L} \right)^2 \right]$$

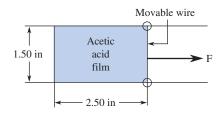
where K = 0.810 lbf. Determine the elastic work (with the appropriate sign) required to stretch the rubber band from an initial length of 2.00 in to a final length of 3.00 in (Figure 4.32).



#### **FIGURE 4.32**

Problem 37.

- **38.\*** A 10.0 cm soap bubble is blown on the end of a large-diameter blowpipe. When the blowpipe end opposite the bubble is uncovered, the surface tension in the soap bubble causes it to collapse, thus sending its contents through the blowpipe and the atmosphere. Estimate the velocity of the air in the blowpipe as the bubble collapses. For the soap bubble,  $\sigma_s = 0.0400 \text{ N/m}$ .
- **39.** At 68.0°F the surface tension of acetic acid is  $1.59 \times 10^{-4}$  lbf/in. A film of acetic acid is maintained on the wire frame as shown in Figure 4.33. Determine the surface tension work done when the wire is moved 1.00 inch in the direction indicated.



#### FIGURE 4.33

Problem 39.

- **40.\*** A 12.0 V automobile battery receives a constant charge from the engine's alternator. The voltage across the terminals is 12.5 V dc, and the current is 9.00 A. Determine the electrical work energy transport rate from the automobile's engine to the battery in both watts and horsepower.
- **41.** A battery powered wheelchair uses a standard 12.0 V automotive lead-acid battery with a capacity of 20.0 A · h. Peukert's law for the discharge of lead-acid batteries is

$$\sigma i^{1.4} = K$$

where  $\sigma$  is the discharge time, *i* is the discharge current, and *K* is a constant that depends on the battery size. The capacity of the battery is given by capacity =  $\sigma i = Ki^{-0.4}$ , and the average voltage during discharge is given by  $\phi = 11.868 - 0.0618i$ .

- a. How much current is drawn from the battery if the torque on the drive shaft is 1.00 ft·lb when it is rotating at 1.00 rev/s?
- **b.** How long will the wheelchair operate with this current drain before the battery is discharged?
- **c.** Evaluate the constant *K* for this battery with this current drain.

- **42.** Determine the electrical current power averaged over one period, *T*, for a sawtooth current waveform passing through a pure resistance *R* described by  $i = i_{max}(t/T)$  for 0 < t < T.
- **43.** In an ac circuit in which a phase angle  $\theta$  exists, the voltage and current are written as

$$\begin{aligned} \varphi &= \varphi_{\max} \cos \left( 2\pi f t \right) \\ i &= i_{\max} \cos \left( 2\pi f t - \theta \right) \end{aligned}$$

Show that the electrical current power averaged over one period (1/f) is

$$(\dot{W})_{\text{electrical avg.}} = -\left(\frac{1}{2}\right)(\phi_{\max})i_{\max}\cos\left(\theta\right) = -\phi_e i_e \cos\left(\theta\right)$$

and thus the average power of any purely reactive  $(\theta = \pi/2)$  circuit consisting entirely of ideal inductors and capacitors is zero. The term cos ( $\theta$ ) is called the *power factor*, and the product  $\phi_e i_e$  is called the *apparent* power. For a purely resistive circuit,  $\theta = 0$  and the average power equals the apparent power. **44.** Show that the polarization work required to charge a parallel

 $_{1}W_{2} = -C\phi^{2}/2$ 

plate capacitor is given by?A3B2 tptxb +2pt?>

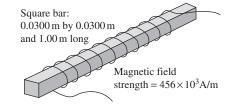
where  $C = \varepsilon_0 \chi_e A/d$  is the capacitance,  $\phi$  is the voltage difference, *A* is the area of the plates, and *d* is their separation distance.

**45.** An electrical capacitor constructed of two parallel conducting plates of area *A*, separated by a distance *d*, has a capacitance *C* given by

$$C = \varepsilon_0 \chi_e A/d$$

where *C* is in faradays (1 F = 1 J/V<sup>2</sup>). Determine the polarization work required to charge an initially discharged 10.0  $\mu$ F parallel plate capacitor when the plates are separated by 5.00 × 10<sup>-3</sup> m of Plexiglas and subjected to a potential difference of 300. V at 27.0°C.

- **46.** A typical storm cloud at an altitude of 3000. ft has a crosssectional area of  $1.00 \times 10^8$  ft<sup>2</sup> and a surface potential relative to the earth of  $1.00 \times 10^8$  V. Determine the amount of electrical energy stored in the cloud by calculating the polarization work required to charge the earth-cloud capacitor.
- **47.\*** A square aluminum bar 0.0300 m on a side and 1.00 m long is wrapped with a current-carrying wire (Figure 4.34). When the current in the wire is turned on, it exposes the aluminum core to a magnetic field strength of  $456 \times 10^3$  A/m. Determine the total magnetic work that occurs when the current is turned on and determine what percentage of this work is associated with the alignment of the aluminum's molecular magnetic dipoles.



# FIGURE 4.34

Problem 47.

**48.\*** A quartz rod 0.0100 m in diameter and 0.100 m long is to be subjected to a magnetic intensity of 10,000. A/m. Determine the

total magnetic work required for this process if the initial magnetic intensity of the rod is zero.

49.\* A Curie substance has a magnetic susceptibility given by

$$\chi_m = C'/T$$

where C' is the Curie constant for the substance and T is its absolute temperature. Determine an expression for the work per unit volume for isothermal material magnetization of a constant volume Curie substance. Evaluate this for

C'' = 153 K, T = 300. K,  $M_1 = 0$ ,  $M_2 = 1000$ . A/m.

- 50.\* The chemical potential of a professor's brain in a single species cranium is constant at -13.2 MJ/kg. Determine the chemical work required to remove 3.77 kg of this valuable substance from the cranium.
- **51.** 2.00 lbm of chemical species A ( $\mu_A = -5700$ . Btu/lbm) is removed from a system while 7.30 lbm of species *B* ( $\mu_B = -3850$  Btu/lbm) and 11.1 lbm of species *C* ( $\mu_C = 1050$  Btu/lbm) are added to the system. Determine the net chemical work involved. Assume constant chemical potentials.
- **52.** If the total internal energy of an adiabatic, stationary, closed system is given by

$$U = -p + \sum \mu_i m_i - f \ell$$

Show that the following formula must hold:

$$- \Psi \, dp + \sum m_i \, d\mu_i - \ell \, df = 0$$

(Hint: Start from the differential form of the energy balance,  $\overline{d}Q - \overline{d}W = dU$  and use Eq. (4.69)).

- **53.** A simple mechanochemical engine operates on the thermodynamic cycle shown in Figure 4.35. The mechanochemical contractile work output ( $fd\ell$ ) comes from a chemical work input ( $\mu dm$ ) due to the aqueous dilution of a single chemical species (i = 1).
  - **a.** Show that the net chemical transport per cycle of this engine is given by

$$(W)_{\text{chemical}} = (\mu_1 - \mu_2)(\Delta m)$$
  
cycle net

where  $\Delta m = m_3 - m_2 = m_4 - m_1$ .

**b.** Write an expression for the work transport energy efficiency of this engine.

- 54. A refrigeration cycle is chosen to maintain a freezer compartment at 10.0°F in a room that is at 90.0°F. If 200. Btu/ min are extracted from the freezer compartment by heat transfer and the freezer is driven by a 1.00 hp electric motor, determine the dimensionless coefficient of performance (COP) of the unit, defined as the cooling rate divided by the input power.
- **55.** An automobile engine produces 127 hp of actual output power. If the friction, heat transfer, and other losses consume 23.0 hp, determine the work transport energy efficiency of this engine.
- **56.**\* 60.0 kW enter a mechanical gearbox at its input shaft but only 55.0 kW exit at its output shaft. Determine its work transport energy efficiency.
- **57.** Find the heat transport rate of energy from a circular pipe with a 2.00 inch outside diameter, 20.0 ft long, and a wall thickness of 0.150 in. The inside and outside surface temperatures of the pipe are 212 and 200.°F, respectively. The pipe is made of carbon steel.
- 58.\* A wall is made up of carbon steel 1.00 cm thick. Determine the conduction heat transport rate per unit area through the wall when the outside temperature is 20.0°C and the inside temperature is −10.0°C.
- **59.** A window consists of a 0.125 in glass pane. Determine the conduction heat transport rate per unit area of window pane when the inside and outside temperatures to be 70.0 and 0.0°F, respectively.
- **60.\*** Find the surface temperature of a bare 40.0 W fluorescent light tube, 3.60 cm in diameter and 1.22 m long in room air at 20.0°C. The convective heat transfer coefficient of the tube is  $4.80 \text{ W/(m}^2 \cdot \text{K})$ .
- 61.\* An experiment has been conducted on a small cylindrical antenna 12.7 mm in diameter and 95.0 mm long. It was heated internally with a 40.0 W electric heater. During the experiment, it was put into a cross flow of air at 26.2°C and 10.0 m/s. Its surface temperature was measured and found to be 127.8°C. Determine the convective heat transfer coefficient for the antenna.
- **62.** An automobile is parked outdoors on a cold evening, when the surrounding air temperature is 35.0°F. The convective heat transfer between the roof of the automobile and the surrounding

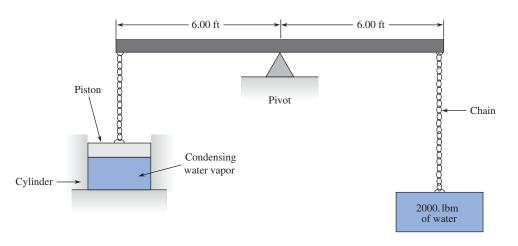


Table 4.12   Problem 67									
Process	Q <sub>cond</sub>	Q <sub>conv</sub>	<b>Q</b> <sub>rad</sub>	W <sub>mech</sub>	Welect	<b>W</b> <sub>magn</sub>	<b>W</b> <sub>chem</sub>	$E_2 - E_1$	
1–2	5	13	-34	45	2	-23	11	?	
2–3	12.3	56.1	121.	0.0	85.0	0.0	?	211.0	
3–1	1.1	-23.3	?	-44.8	89.9	-47.3	14.2	0.0	

air is 1.50 Btu/( $h \cdot ft^2 \cdot R$ ). The night sky is cloudless and forms a black body at a temperature of  $-30.0^\circ F$ . By performing a convective-radiation balance on the roof, determine

a. The roof temperature.

**b**. Whether or not frost will form on the roof (and why).

- **63.\*** Determine the radiation heat transfer rate per unit area between an infant at 37.0°C in a crib and a nearby window at -10.0°C in the winter. The view factor between the infant and the window is 0.310.
- **64.** Determine the radiation heat transfer rate per unit area from a nuclear fireball at 10,000.°F and a nearby building at 70.0°F covered with white paint. The view factor between the building and the fireball is 0.0100.
- 65. Define the following terms:
  - a. Adiabatic.
  - b. Mechanical work.
  - **c.** Reversible.
  - d. The state postulate.
- **66.** Define the following terms:
  - a. Aergonic.
  - b. The local equilibrium postulate.
  - c. Enthalpy.
  - d. Work efficiency.
- **67.\*** A closed system undergoes a cycle made up of three processes. Fill in the missing data in Table 4.12. All the values are in kilojoules.

# **Computer Problems**

The following computer assignments are designed to be carried out on a personal computer using a spreadsheet or equation solver. They are exercises that use some of the basic formulae of Chapter 4. They may be used as part of a weekly homework assignment.

- **68.** Develop a program that calculates the work transport for an ideal gas undergoing a polytropic moving boundary process. Have the user input all necessary data from the keyboard by responding to properly worded screen prompts. Make sure that units are specified when requesting user data input. Output the polytropic work and all the input data (with corresponding units).
- **69.** Develop a program that calculates the work transport for a Hookean elastic solid. Have the user input all necessary data from the keyboard by responding to properly worded screen prompts. Make sure that units are specified when requesting user data input. Output the elastic work and all the input data (with corresponding units).
- **70.** Develop a program that determines the work transport in a constant volume magnetization process. Have the user input all necessary data from the keyboard by responding to properly worded screen prompts. Make sure that units are specified when requesting user data input. Output the magnetic work, the work

of magnetization of the exposed material, and all the input data (with corresponding units).

- **71.** Develop a program that determines the chemical work transport for a system with constant chemical potentials. Have the user input all the  $\mu_i$  and the initial and final  $m_i$  from the keyboard by responding to properly worded screen prompts. Make sure that units are specified when requesting user data input. Output the chemical work (with corresponding units).
- **72.** Develop a program that determines the total heat transfer rate from the sum of one or more of the three heat transport modes (conduction, convection, and radiation). Have the user select from a menu which heat transport mode or combination of heat transport modes he or she wishes to use. Then have the user input all necessary data from the keyboard by responding to properly worded screen prompts. Output the heat transport rate and all input data (with corresponding units).

# **Create and Solve Problems**

Engineering education tends to focus only on the process of solving problems. It ignores teaching the process of formulating solvable problems. However, working engineers are never given a well-phrased problem statement to solve. Instead, they need to react to situational information and organize it into a structure that can be solved using the methods learned in college.

Also, if you see how problems are written (created), then you have a better chance of mastering the solution technique and of understanding how to structure information as a working engineer into solvable situations. These "Create and Solve" problems are designed to help you learn how to formulate solvable thermodynamics problems from engineering data. Since you provide the numerical values for some of the variables, these problems have no unique solutions. Their solutions depend on the assumptions you need to make and how you set them up to create a solvable problem.

73.\* You are a design engineer working on a robotic system. The robot contains an imbedded circuit board that draws 30.0 mA at 5.00 V. Someone mentions that the circuit board might overheat during its 30 min. operating cycle. Write and solve a problem that provides (a) the heat generation rate of the circuit board, and (b) the temperature of the circuit board if it is insulated and operated for 30.0 minutes. Choose relevant values for the necessary variables. Hint: Your problem statement might read something like this:

An insulated circuit board draws 30.0 mA at 5.00 V. Determine its heat generation rate and its temperature after 30.0 min of operation. The board has a mass of  $1.00 \times 10^{-3}$  kg and its specific heat is 0.500 kJ/kg·K.

Now you have to solve your problem to determine the answers to (a) and (b).

- **74.** You are designing a new mechanical transmission large rock crusher used in the mining industry. The transmission is driven by a 300. hp engine but transmits only 290. hp to the rock crusher. You need to prevent the transmission from overheating, so how much cooling is needed to keep it at ambient temperature?
- **75.\*** Dave, your boss, wants you to estimate the amount of heat that has to be removed from an iron ingot to cool it from 900°C to 150°C. Make this request into a thermodynamic problem statement and solve it.
- 76.\* You are a new engineer at a company that manufactures gas-filled shock absorbers for racing cars. The chief engineer wants to understand the relation between the gas pressure inside the shock absorber and the compression of the gas. The shock absorbers are essentially piston-cylinder devices that are initially filled with nitrogen gas at 0.345 MPa and 20.0°F. When the piston compresses the gas by 20%, the pressure increases to 0.414 MPa. You think this is a polytropic compression process. Write and solve a thermodynamics problem to determine the polytropic exponent for this process.