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Computational Finance
Using C and C#

George Levy
To my parents Paul and Paula
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Preface

This book builds on the author’s previous book Computational Finance: Numerical Methods for Pricing Financial Instruments, which contained information on pricing equity options using C code. The current book covers the following instrument types:

- Equity derivatives
- Interest rate derivatives
- Foreign exchange derivatives
- Credit derivatives

There is also an extensive final chapter which demonstrates how a C-based analytics pricing library can be used by C# portfolio valuation software. In addition this application:

- illustrates the use of C# dictionaries, abstract classes and .NET InteropServices
- permits the reader to value bespoke portfolios
- allows market data to be specified via a configuration file
- contains a generic basket pricer for which the reader can specify the payoff function
- can be freely downloaded for use by the reader.

The current book also contains increased coverage of stochastic processes, Ito calculus and Monte Carlo simulation. These topics are supported by practical applications and solved example problems.

In addition the Numerical Algorithms Group (NAG) have allowed readers to enjoy an extended trial licence for the NAG C library and associated financial routines from the following url: www.nag.co.uk-market/elsevier_glevy. The NAG C library may be called into C# and provides a large suite of mathematical routines addressing many areas covered in this book (random numbers, statistical distributions, option pricing, correlation and covariance matrices etc.).

Computational Finance Using C and C# also includes supporting software that may be downloaded for free. The software consists of executable files, configuration files and results files. With these files the user can run the example portfolio application in Chapter 8 and change the portfolio composition and the attributes of the deals.

Additional upgrade software is available for purchase with Computational Finance Using C and C#. The software includes:

- Code to run all the C, C# and Excel examples in the book
• Complete C source code for the Analytics_Mathlib math library that is used in the book
• C# source code, market data and portfolio files for the portfolio application described in Chapter 8

All the C/C# software in the book can be compiled using either Visual Studio .NET 2005, or the freely available Microsoft Visual C#/C++ Express Editions.

I would like to take this opportunity of thanking my wife Kathy for her support.

In addition I am grateful to Karen Maloney of Elsevier for her patience with regard to the book’s delivery date, and Dr. Stephen Satchell of Trinity College Cambridge for allowing me the opportunity to write a sequel.

George Levy
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