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Stresses in Pressure Vessels

DESIGN PHILOSOPHY

In general, pressure vessels designed in accordance with the ASME Code, Section VIII, Division 1, are designed by rules and do not require a detailed evaluation of all stresses. It is recognized that high localized and secondary bending stresses may exist but are allowed for by use of a higher safety factor and design rules for details. It is required, however, that all loadings (the forces applied to a vessel or its structural attachments) must be considered. (See Reference 1, Para. UG-22.)

While the Code gives formulas for thickness and stress of basic components, it is up to the designer to select appropriate analytical procedures for determining stress due to other loadings. The designer must also select the most probable combination of simultaneous loads for an economical and safe design.

The Code establishes allowable stresses by stating in Para. UG-23(c) that the maximum general primary membrane stress must be less than allowable stresses outlined in material sections. Further, it states that the maximum primary membrane stress *plus* primary bending stress may not exceed 1.5 times the allowable stress of the material sections. In other sections, specifically Paras. 1-5(e) and 2-8, higher allowable stresses are permitted if appropriate analysis is made. These higher allowable stresses clearly indicate that different stress levels for different stress categories are acceptable.

It is general practice when doing more detailed stress analysis to apply higher allowable stresses. In effect, the detailed evaluation of stresses permits substituting knowledge of localized stresses and the use of higher allowables in place of the larger factor of safety used by the Code. This higher safety factor really reflected lack of knowledge about actual stresses.

A calculated value of stress means little until it is associated with its location and distribution in the vessel and with the type of loading that produced it. Different types of stress have different degrees of significance.

The designer must familiarize himself with the various types of stress and loadings in order to accurately apply the results of analysis. The designer must also consider some adequate stress or failure theory in order to combine stresses and set allowable stress limits. It is against this failure mode that he must compare and interpret stress values, and define how the stresses in a component react and contribute to the strength of that part.

The following sections will provide the fundamental knowledge for applying the results of analysis. The topics covered in Chapter 1 form the basis by which the rest of the book is to be used. A section on special problems and considerations is included to alert the designer to more complex problems that exist.

STRESS ANALYSIS

Stress analysis is the determination of the relationship between external forces applied to a vessel and the corresponding stress. The emphasis of this book is not how to do stress analysis in particular, but rather how to analyze vessels and their component parts in an effort to arrive at an economical and safe design—the difference being that we analyze stresses where necessary to determine thickness of material and sizes of members. We are not so concerned with building mathematical models as with providing a step-by-step approach to the design of ASME Code vessels. It is not necessary to find every stress but rather to know the governing stresses and how they relate to the vessel or its respective parts, attachments, and supports.

The starting place for stress analysis is to determine all the design conditions for a given problem and then determine all the related external forces. We must then relate these external forces to the vessel parts which must resist them to find the corresponding stresses. By isolating the causes (loadings), the effects (stress) can be more accurately determined.

The designer must also be keenly aware of the types of loads and how they relate to the vessel as a whole. Are the effects long or short term? Do they apply to a localized portion of the vessel or are they uniform throughout?

How these stresses are interpreted and combined, what significance they have to the overall safety of the vessel, and what allowable stresses are applied will be determined by three things:

1. The strength/failure theory utilized.
2. The types and categories of loadings.
3. The hazard the stress represents to the vessel.

Membrane Stress Analysis

Pressure vessels commonly have the form of spheres, cylinders, cones, ellipsoids, tori, or composites of these. When the thickness is small in comparison with other dimensions ($R_m/t > 10$), vessels are referred to as membranes and the associated stresses resulting from the contained pressure are called membrane stresses. These membrane stresses are average tension or compression stresses. They are assumed to be uniform across the vessel wall and act tangentially to its surface. The membrane or wall is assumed to offer no resistance to bending. When the wall offers resistance to bending, bending stresses occur in addition to membrane stresses.

In a vessel of complicated shape subjected to internal pressure, the simple membrane-stress concepts do not suffice to give an adequate idea of the true stress situation. The types of heads closing the vessel, effects of supports, variations in thickness and cross section, nozzles, external attachments, and overall bending due to weight, wind, and seismic activity all cause varying stress distributions in the vessel. Deviations from a true membrane shape set up bending in the vessel wall and cause the direct loading to vary from point to point. The direct loading is diverted from the more flexible to the more rigid portions of the vessel. This effect is called "stress redistribution."

In any pressure vessel subjected to internal or external pressure, stresses are set up in the shell wall. The state of stress is triaxial and the three principal stresses are:

$\sigma_x =$ longitudinal/meridional stress
$\sigma_\phi =$ circumferential/latitudinal stress
$\sigma_r =$ radial stress

In addition, there may be bending and shear stresses. The radial stress is a direct stress, which is a result of the pressure acting directly on the wall, and causes a compressive stress equal to the pressure. In thin-walled vessels this stress is so small compared to the other "principal" stresses that it is generally ignored. Thus we assume for purposes of analysis that the state of stress is biaxial. This greatly simplifies the method of combining stresses in comparison to triaxial stress states. For thickwalled vessels ($R_m/t < 10$), the radial stress cannot be ignored and formulas are quite different from those used in finding "membrane stresses" in thin shells.

Since ASME Code, Section VIII, Division 1, is basically for design by rules, a higher factor of safety is used to allow for the "unknown" stresses in the vessel. This higher safety factor, which allows for these unknown stresses, can impose a penalty on design but requires much less analysis. The design techniques outlined in this text are a compromise between finding all stresses and utilizing minimum code formulas. This additional knowledge of stresses warrants the use of higher allowable stresses in some cases, while meeting the requirements that all loadings be considered.

In conclusion, "membrane stress analysis" is not completely accurate but allows certain simplifying assumptions to be made while maintaining a fair degree of accuracy. The main simplifying assumptions are that the stress is biaxial and that the stresses are uniform across the shell wall. For thin-walled vessels these assumptions have proven themselves to be reliable. No vessel meets the criteria of being a true membrane, but we can use this tool with a reasonable degree of accuracy.

STRESS/FAILURE THEORIES

As stated previously, stresses are meaningless until compared to some stress/failure theory. The significance of a given stress must be related to its location in the vessel and its bearing on the ultimate failure of that vessel. Historically, various "theories" have been derived to combine and measure stresses against the potential failure mode. A number of stress theories, also called "yield criteria," are available for describing the effects of combined stresses. For purposes of this book, as these failure theories apply to pressure vessels, only two theories will be discussed. They are the "maximum stress theory" and the "maximum shear stress theory."

Maximum Stress Theory

This theory is the oldest, most widely used and simplest to apply. Both ASME Code, Section VIII, Division 1, and Section I use the maximum stress theory as a basis for design. This theory simply asserts that the breakdown of material depends only on the numerical magnitude of the maximum principal or normal stress. Stresses in the other directions are disregarded. Only the maximum principal stress must be determined to apply this criterion. This theory is used for biaxial states of stress assumed in a thin-walled pressure vessel. As will be shown later it is unconservative in some instances and requires a higher safety factor for its use. While the maximum stress theory does accurately predict failure in brittle materials, it is not always accurate for ductile materials. Ductile materials often fail along lines 45° to the applied force by shearing, long before the tensile or compressive stresses are maximum.

This theory can be illustrated graphically for the four states of biaxial stress shown in Figure 1-1.

It can be seen that uniaxial tension or compression lies on the two axes. Inside the box (outer boundaries) is the elastic range of the material. Yielding is predicted for stress combinations by the outer line.

Maximum Shear Stress Theory

This theory asserts that the breakdown of material depends only on the maximum shear stress attained in an element. It assumes that yielding starts in planes of maximum shear stress. According to this theory, yielding will start at a point when the maximum shear stress at that point reaches one-half of the the uniaxial yield strength, F_y. Thus for a biaxial state of stress where $\sigma_1 > \sigma_2$, the maximum shear stress will be $(\sigma_1 - \sigma_2)/2$.

Yielding will occur when

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{F_y}{2}$$

Both ASME Code, Section VIII, Division 2 and ASME Code, Section III, utilize the maximum shear stress criterion. This theory closely approximates experimental results and is also easy to use. This theory also applies to triaxial states of stress. In a triaxial stress state, this theory predicts that yielding will occur whenever one-half the algebraic difference between the maximum and minimum stress is equal to one-half the yield stress. Where $\sigma_1 > \sigma_2 > \sigma_3$, the maximum shear stress is $(\sigma_1 - \sigma_3)/2$.

Yielding will begin when

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{F_y}{2}$$

This theory is illustrated graphically for the four states of biaxial stress in Figure 1-2.

A comparison of Figure 1-1 and Figure 1-2 will quickly illustrate the major differences between the two theories. Figure 1-2 predicts yielding at earlier points in Quadrants II and IV. For example, consider point B of Figure 1-2. It shows $\sigma_2 = (-)\sigma_1$; therefore the shear stress is equal to $\sigma_2 - (-\sigma_1)/2$, which equals $\sigma_2 + \sigma_1/2$ or one-half the stress

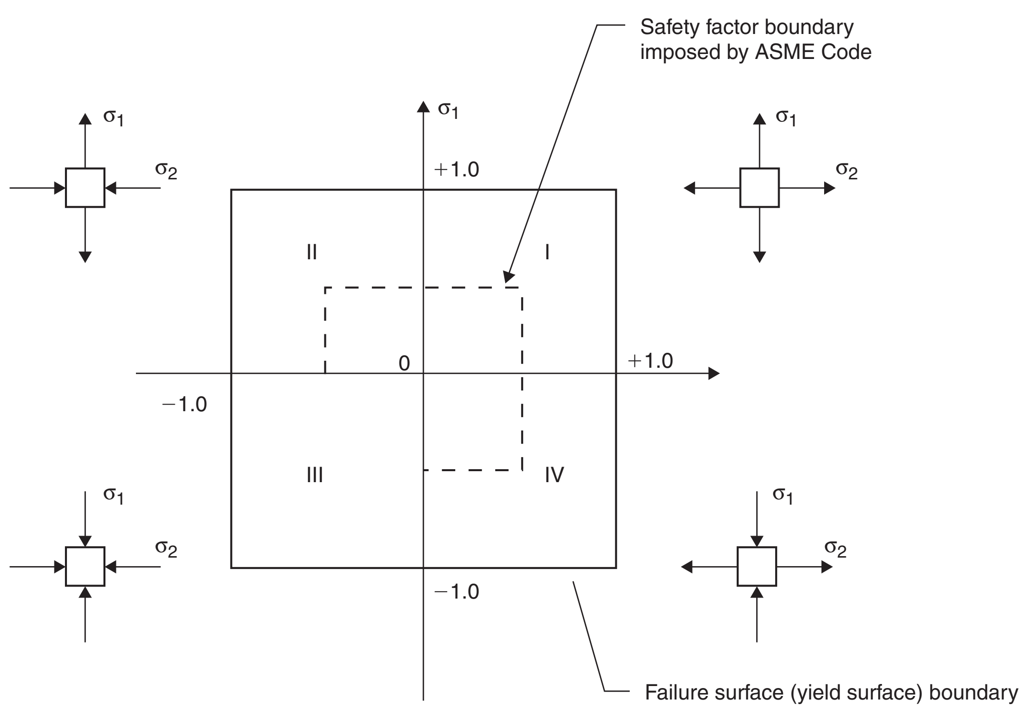

Figure 1-1. Graph of maximum stress theory. Quadrant I: biaxial tension; Quadrant II: tension; Quadrant III: biaxial compression; Quadrant IV: compression.

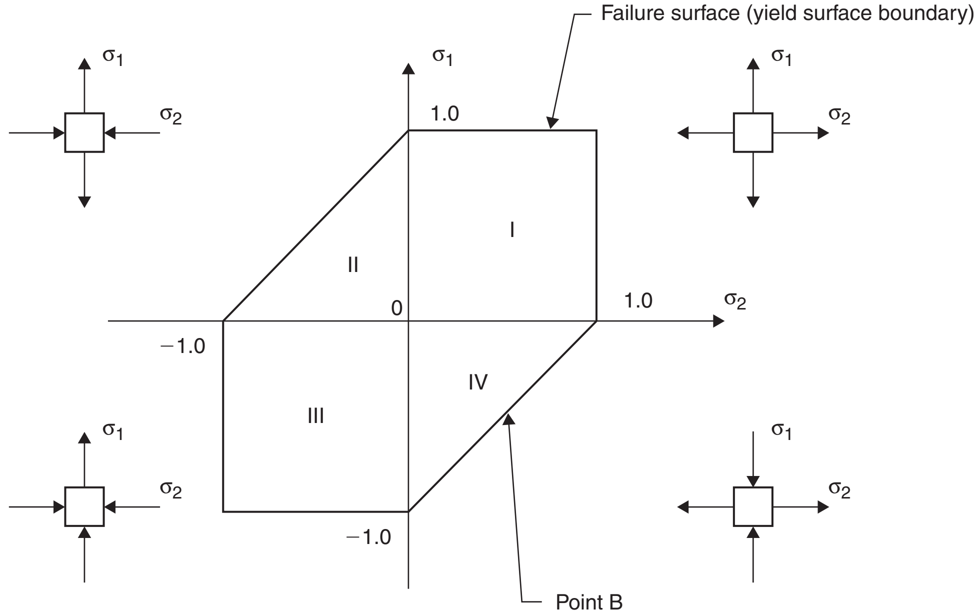

Figure 1-2. Graph of maximum shear stress theory.

which would cause yielding as predicted by the maximum stress theory!

Comparison of the Two Theories

Both theories are in agreement for uniaxial stress or when one of the principal stresses is large in comparison to the others. The discrepancy between the theories is greatest when both principal stresses are numerically equal.

For simple analysis upon which the thickness formulas for ASME Code, Section I or Section VIII, Division 1, are based, it makes little difference whether the maximum stress theory or maximum shear stress theory is used. For example, according to the maximum stress theory, the controlling stress governing the thickness of a cylinder is σ_ϕ, circumferential stress, since it is the largest of the three principal stresses. According to the maximum shear stress theory, the controlling stress would be one-half the algebraic difference between the maximum and minimum stress:

- The maximum stress is the circumferential stress, σ_ϕ

  $\sigma_\phi = PR/t$

- The minimum stress is the radial stress, σ_r

  $\sigma_r = -P$

Therefore, the maximum shear stress is:

$$\frac{\sigma_\phi - \sigma_r}{2}$$

ASME Code, Section VIII, Division 2, and Section III use the term "stress intensity," which is defined as twice the maximum shear stress. Since the shear stress is compared to one-half the yield stress only, "stress intensity" is used for comparison to allowable stresses or ultimate stresses. To define it another way, yielding begins when the "stress intensity" exceeds the yield strength of the material.

In the preceding example, the "stress intensity" would be equal to $\sigma_\phi - \sigma_r$. And

$$\sigma_\phi - \sigma_r = PR/t - (-P) = PR/t + P$$

For a cylinder where P = 300 psi, R = 30 in., and t = .5 in., the two theories would compare as follows:

- *Maximum stress theory*

  $\sigma = \sigma_\phi = PR/t = 300(30)/.5 = 18{,}000$ psi

- *Maximum shear stress theory*

  $\sigma = PR/t + P = 300(30)/.5 + 300 = 18{,}300$ psi

Two points are obvious from the foregoing:

1. For thin-walled pressure vessels, both theories yield approximately the same results.
2. For thin-walled pressure vessels the radial stress is so small in comparison to the other principal stresses that it can be ignored and a state of biaxial stress is assumed to exist.

For thick-walled vessels ($R_m/t < 10$), the radial stress becomes significant in defining the ultimate failure of the vessel. The maximum stress theory is unconservative for designing these vessels. For this reason, this text has limited its application to thin-walled vessels where a biaxial state of stress is assumed to exist.

FAILURES IN PRESSURE VESSELS

Vessel failures can be grouped into four major categories, which describe *why* a vessel failure occurs. Failures can also be grouped into types of failures, which describe *how* the failure occurs. Each failure has a why and how to its history. It may have failed *through* corrosion fatigue *because* the wrong material was selected! The designer must be as familiar with categories and types of failure as with categories and types of stress and loadings. Ultimately they are all related.

Categories of Failures

1. *Material*—Improper selection of material; defects in material.
2. *Design*—Incorrect design data; inaccurate or incorrect design methods; inadequate shop testing.
3. *Fabrication*—Poor quality control; improper or insufficient fabrication procedures including welding; heat treatment or forming methods.
4. *Service*—Change of service condition by the user; inexperienced operations or maintenance personnel; upset conditions. Some types of service which require special attention both for selection of material, design details, and fabrication methods are as follows:
   a. Lethal
   b. Fatigue (cyclic)
   c. Brittle (low temperature)
   d. High temperature
   e. High shock or vibration
   f. Vessel contents
      - Hydrogen
      - Ammonia
      - Compressed air
      - Caustic
      - Chlorides

Types of Failures

1. *Elastic deformation*—Elastic instability or elastic buckling, vessel geometry, and stiffness as well as properties of materials are protection against buckling.
2. *Brittle fracture*—Can occur at low or intermediate temperatures. Brittle fractures have occurred in vessels made of low carbon steel in the 40°–50°F range during hydrotest where minor flaws exist.
3. *Excessive plastic deformation*—The primary and secondary stress limits as outlined in ASME Section VIII, Division 2, are intended to prevent excessive plastic deformation and incremental collapse.
4. *Stress rupture*—Creep deformation as a result of fatigue or cyclic loading, i.e., progressive fracture. Creep is a time-dependent phenomenon, whereas fatigue is a cycle-dependent phenomenon.
5. *Plastic instability*—Incremental collapse; incremental collapse is cyclic strain accumulation or cumulative cyclic deformation. Cumulative damage leads to instability of vessel by plastic deformation.
6. *High strain*—Low cycle fatigue is strain-governed and occurs mainly in lower-strength/high-ductile materials.
7. *Stress corrosion*—It is well known that chlorides cause stress corrosion cracking in stainless steels; likewise caustic service can cause stress corrosion cracking in carbon steels. Material selection is critical in these services.
8. *Corrosion fatigue*—Occurs when corrosive and fatigue effects occur simultaneously. Corrosion can reduce fatigue life by pitting the surface and propagating cracks. Material selection and fatigue properties are the major considerations.

In dealing with these various modes of failure, the designer must have at his disposal a picture of the state of stress in the various parts. It is against these failure modes that the designer must compare and interpret stress values. But setting allowable stresses is not enough! For elastic instability one must consider geometry, stiffness, and the properties of the material. Material selection is a major consideration when related to the type of service. Design details and fabrication methods are as important as "allowable stress" in design of vessels for cyclic service. The designer and all those persons who ultimately affect the design must have a clear picture of the conditions under which the vessel will operate.

LOADINGS

Loadings or forces are the "causes" of stresses in pressure vessels. These forces and moments must be isolated both to determine *where* they apply to the vessel and *when* they apply to a vessel. Categories of loadings define where these forces are applied. Loadings may be applied over a large portion (general area) of the vessel or over a local area of the vessel. Remember both *general* and *local* loads can produce membrane and bending stresses. These stresses are additive and define the overall state of stress in the vessel or component. Stresses from local loads must be added to stresses from general loadings. These combined stresses are then compared to an allowable stress.

Consider a pressurized, vertical vessel bending due to wind, which has an inward radial force applied locally. The effects of the pressure loading are longitudinal and circumferential tension. The effects of the wind loading are longitudinal tension on the windward side and longitudinal compression on the leeward side. The effects of the local inward radial load are some local membrane stresses and local bending stresses. The local stresses would be both circumferential and longitudinal, tension on the inside surface of the vessel, and compressive on the outside. Of course the steel at any given point only sees a certain level of stress or the combined effect. It is the designer's job to combine the stresses from the various loadings to arrive at the worst probable combination of stresses, combine them using some failure theory, and compare the results to an acceptable stress level to obtain an economical and safe design.

This hypothetical problem serves to illustrate how categories and types of loadings are related to the stresses they produce. The stresses applied more or less *continuously* and *uniformly* across an entire section of the vessel are primary stresses.

The stresses due to pressure and wind are primary membrane stresses. These stresses should be limited to the code allowable. These stresses would cause the bursting or collapse of the vessel if allowed to reach an unacceptably high level.

On the other hand, the stresses from the inward radial load could be either a primary local stress or secondary stress. It is a primary local stress if it is produced from an unrelenting load or a secondary stress if produced by a relenting load. Either stress may cause local deformation but will not in and of itself cause the vessel to fail. If it is a primary stress, the stress will be redistributed; if it is a secondary stress, the load will relax once slight deformation occurs.

Also be aware that this is only true for ductile materials. In brittle materials, there would be no difference between primary and secondary stresses. If the material cannot yield to reduce the load, then the definition of secondary stress does not apply! Fortunately current pressure vessel codes require the use of ductile materials.

This should make it obvious that the type and category of loading will determine the type and category of stress. This will be expanded upon later, but basically each combination of stresses (stress categories) will have different allowables, i.e.:

- Primary stress: $P_m < SE$
- Primary membrane local (P_L):

$$P_L = P_m + P_L < 1.5\ SE$$
$$P_L = P_m + Q_m < 1.5\ SE$$

- Primary membrane + secondary (Q):

$$P_m + Q < 3\ SE$$

But what if the loading was of relatively short duration? This describes the "type" of loading. Whether a loading is steady, more or less continuous, or nonsteady, variable, or temporary will also have an effect on what level of stress will be acceptable. If in our hypothetical problem the loading had been pressure + seismic + local load, we would have a different case. Due to the relatively short duration of seismic loading, a higher "temporary" allowable stress would be acceptable. The vessel doesn't have to operate in an earthquake all the time. On the other hand, it also shouldn't fall down in the event of an earthquake! Structural designs allow a one-third increase in allowable stress for seismic loadings for this reason.

For *steady loads*, the vessel must support these loads more or less continuously during its useful life. As a result, the stresses produced from these loads must be maintained to an acceptable level.

For *nonsteady loads*, the vessel may experience some or all of these loadings at various times but not all at once and not more or less continuously. Therefore a temporarily higher stress is acceptable.

For *general loads* that apply more or less uniformly across an entire section, the corresponding stresses must be lower, since the entire vessel must support that loading.

For *local loads,* the corresponding stresses are confined to a small portion of the vessel and normally fall off rapidly in distance from the applied load. As discussed previously, pressurizing a vessel causes bending in certain components. But it doesn't cause the entire vessel to bend. The results are not as significant (except in cyclic service) as those caused by general loadings. Therefore a slightly higher allowable stress would be in order.

Loadings can be outlined as follows:

A. *Categories of loadings*
1. *General loads*—Applied more or less continuously across a vessel section.
   a. Pressure loads—Internal or external pressure (design, operating, hydrotest, and hydrostatic head of liquid).
   b. Moment loads—Due to wind, seismic, erection, transportation.
   c. Compressive/tensile loads—Due to dead weight, installed equipment, ladders, platforms, piping, and vessel contents.
   d. Thermal loads—Hot box design of skirthead attachment.
2. *Local loads*—Due to reactions from supports, internals, attached piping, attached equipment, i.e., platforms, mixers, etc.
   a. Radial load—Inward or outward.
   b. Shear load—Longitudinal or circumferential.
   c. Torsional load.
   d. Tangential load.
   e. Moment load—Longitudinal or circumferential.
   f. Thermal load.

B. *Types of loadings*
1. *Steady loads*—Long-term duration, continuous.
   a. Internal/external pressure.
   b. Dead weight.
   c. Vessel contents.
   d. Loadings due to attached piping and equipment.
   e. Loadings to and from vessel supports.
   f. Thermal loads.
   g. Wind loads.
2. *Nonsteady loads*—Short-term duration; variable.
   a. Shop and field hydrotests.
   b. Earthquake.
   c. Erection.
   d. Transportation.
   e. Upset, emergency.
   f. Thermal loads.
   g. Start up, shut down.

STRESS

ASME Code, Section VIII, Division 1 vs. Division 2

ASME Code, Section VIII, Division 1 does not explicitly consider the effects of combined stress. Neither does it give detailed methods on how stresses are combined. ASME Code, Section VIII, Division 2, on the other hand, provides specific guidelines for stresses, how they are combined, and allowable stresses for categories of combined stresses. Division 2 is design by analysis whereas Division 1 is design by rules. Although stress analysis as utilized by Division 2 is beyond the scope of this text, the use of stress categories, definitions of stress, and allowable stresses is applicable.

Division 2 stress analysis considers all stresses in a triaxial state combined in accordance with the maximum shear stress theory. Division 1 and the procedures outlined in this book consider a biaxial state of stress combined in accordance with the maximum stress theory. Just as you would not design a nuclear reactor to the rules of Division 1, you would not design an air receiver by the techniques of Division 2. Each has its place and applications. The following discussion on categories of stress and allowables will utilize information from Division 2, which can be applied in general to all vessels.

Types, Classes, and Categories of Stress

The shell thickness as computed by Code formulas for internal or external pressure alone is often not sufficient to withstand the combined effects of all other loadings. Detailed calculations consider the effects of each loading separately and then must be combined to give the total state of stress in that part. The stresses that are present in pressure vessels are separated into various *classes* in accordance with the *types* of loads that produced them, and the hazard they represent to the vessel. Each class of stress must be maintained at an acceptable level and the combined total stress must be kept at another acceptable level. The combined stresses due to a combination of loads acting simultaneously are called stress categories. Please note that this terminology differs from that given in Division 2, but is clearer for the purposes intended here.

Classes of stress, categories of stress, and allowable stresses are based on the type of loading that produced them *and* on the hazard they represent to the structure. Unrelenting loads produce primary stresses. Relenting loads (self-limiting) produce secondary stresses. General loadings produce primary membrane and bending stresses. Local loads produce local membrane and bending stresses. Primary stresses must be kept lower than secondary stresses.

Primary plus secondary stresses are allowed to be higher and so on. Before considering the combination of stresses (categories), we must first define the various *types* and *classes* of stress.

Types of Stress

There are many names to describe types of stress. Enough in fact to provide a confusing picture even to the experienced designer. As these stresses apply to pressure vessels, we group all types of stress into three major classes of stress, and subdivision of each of the groups is arranged according to their effect on the vessel. The following list of stresses describes types of stress without regard to their effect on the vessel or component. They define a direction of stress or relate to the application of the load.

1. Tensile
2. Compressive
3. Shear
4. Bending
5. Bearing
6. Axial
7. Discontinuity
8. Membrane
9. Principal
10. Thermal
11. Tangential
12. Load induced
13. Strain induced
14. Circumferential
15. Longitudinal
16. Radial
17. Normal

Classes of Stress

The foregoing list provides examples of types of stress. It is, however, too general to provide a basis with which to combine stresses or apply allowable stresses. For this purpose, new groupings called *classes* of stress must be used. Classes of stress are defined by the type of loading which produces them and the hazard they represent to the vessel.

1. *Primary stress*
   a. General:
      - Primary general membrane stress, P_m
      - Primary general bending stress, P_b
   b. Primary local stress, P_L
2. *Secondary stress*
   a. Secondary membrane stress, Q_m
   b. Secondary bending stress, Q_b
3. *Peak stress*, *F*

Definitions and examples of these stresses follow.

Primary general stress. These stresses act over a full cross section of the vessel. They are produced by mechanical loads (load induced) and are the most hazardous of all types of stress. The basic characteristic of a primary stress is that it is not self-limiting. Primary stresses are generally due to internal or external pressure or produced by sustained external forces and moments. Thermal stresses are never classified as primary stresses.

Primary general stresses are divided into membrane and bending stresses. The need for dividing primary general stress into membrane and bending is that the calculated value of a primary bending stress may be allowed to go higher than that of a primary membrane stress. Primary stresses that exceed the yield strength of the material can cause failure or gross distortion. Typical calculations of primary stress are:

$$\frac{PR}{t}, \frac{F}{A}, \frac{MC}{I}, \quad \text{and} \quad \frac{TC}{J}$$

Primary general membrane stress, P_m. This stress occurs across the entire cross section of the vessel. It is remote from discontinuities such as head-shell intersections, cone-cylinder intersections, nozzles, and supports. Examples are:

a. Circumferential and longitudinal stress due to pressure.
b. Compressive and tensile axial stresses due to wind.
c. Longitudinal stress due to the bending of the horizontal vessel over the saddles.
d. Membrane stress in the center of the flat head.
e. Membrane stress in the nozzle wall within the area of reinforcement due to pressure or external loads.
f. Axial compression due to weight.

Primary general bending stress, P_b. Primary bending stresses are due to sustained loads and are capable of causing collapse of the vessel. There are relatively few areas where primary bending occurs:

a. Bending stress in the center of a flat head or crown of a dished head.
b. Bending stress in a shallow conical head.
c. Bending stress in the ligaments of closely spaced openings.

Local primary membrane stress, P_L. Local primary membrane stress is not technically a classification of stress but a stress category, since it is a combination of two stresses. The combination it represents is primary membrane stress, P_m, plus secondary membrane stress, Q_m, produced from sustained loads. These have been grouped together in order to limit the allowable stress for this particular combination to a level lower than allowed for other primary and secondary stress applications. It was felt that local stress from sustained (unrelenting) loads presented a great enough hazard for the combination to be "classified" as a primary stress.

A local primary stress is produced either by design pressure alone or by other mechanical loads. Local primary stresses have some self-limiting characteristics like secondary stresses. Since they are localized, once the yield strength of the material is reached, the load is redistributed to stiffer portions of the vessel. However, since any deformation associated with yielding would be unacceptable, an allowable stress lower than secondary stresses is assigned. The basic difference between a primary local stress and a secondary stress is that a primary local stress is produced by a load that is unrelenting; the stress is just redistributed. In a secondary stress, yielding relaxes the load and is truly self-limiting. The ability of primary local stresses to redistribute themselves after the yield strength is attained locally provides a safety-valve effect. Thus, the higher allowable stress applies only to a local area.

Primary local membrane stresses are a combination of membrane stresses only. Thus only the "membrane" stresses from a local load are combined with primary general membrane stresses, not the bending stresses. The bending stresses associated with a local loading are secondary stresses. Therefore, the membrane stresses from a WRC-107-type analysis must be broken out separately and combined with primary general stresses. The same is true for discontinuity membrane stresses at head-shell junctures, cone-cylinder junctures, and nozzle-shell junctures. The bending stresses would be secondary stresses.

Therefore, $P_L = P_m + Q_m$, where Q_m is a local stress from a sustained or unrelenting load. Examples of primary local membrane stresses are:

a. P_m + membrane stresses at local discontinuities:
   1. Head-shell juncture
   2. Cone-cylinder juncture
   3. Nozzle-shell juncture
   4. Shell-flange juncture
   5. Head-skirt juncture
   6. Shell-stiffening ring juncture
b. P_m + membrane stresses from local sustained loads:
   1. Support lugs
   2. Nozzle loads
   3. Beam supports
   4. Major attachments

Secondary stress. The basic characteristic of a secondary stress is that it is self-limiting. As defined earlier, this means that local yielding and minor distortions can satisfy the conditions which caused the stress to occur. Application of a secondary stress cannot cause structural failure due to the restraints offered by the body to which the part is attached. Secondary mean stresses are developed at the junctions of major components of a pressure vessel. Secondary mean stresses are also produced by sustained loads other than internal or external pressure. Radial loads on nozzles produce secondary mean stresses in the shell at the junction of the nozzle. Secondary stresses are strain-induced stresses.

Discontinuity stresses are only considered as secondary stresses if their extent along the length of the shell is limited. Division 2 imposes the restriction that the length over which the stress is secondary is $\sqrt{R_m t}$. Beyond this distance, the stresses are considered as primary mean stresses. In a cylindrical vessel, the length $\sqrt{R_m t}$ represents the length over which the shell behaves as a ring.

A further restriction on secondary stresses is that they may not be closer to another gross structural discontinuity than a distance of $2.5\sqrt{R_m t}$. This restriction is to eliminate the additive effects of edge moments and forces.

Secondary stresses are divided into two additional groups, membrane and bending. Examples of each are as follows:

Secondary membrane stress, Q_m.

a. Axial stress at the juncture of a flange and the hub of the flange.
b. Thermal stresses.
c. Membrane stress in the knuckle area of the head.
d. Membrane stress due to local relenting loads.

Secondary bending stress, Q_b.

a. Bending stress at a gross structural discontinuity: nozzles, lugs, etc. (relenting loadings only).
b. The nonuniform portion of the stress distribution in a thick-walled vessel due to internal pressure.
c. The stress variation of the radial stress due to internal pressure in thick-walled vessels.
d. Discontinuity stresses at stiffening or support rings.

Note: For b and c it is necessary to subtract out the average stress which is the primary stress. Only the varying part of the stress distribution is a secondary stress.

Peak stress, F. Peak stresses are the additional stresses due to stress intensification in highly localized areas. They apply to both sustained loads and self-limiting loads. There are no significant distortions associated with peak stresses. Peak stresses are additive to primary and secondary stresses present at the point of the stress concentration. Peak stresses are only significant in fatigue conditions or brittle materials. Peak stresses are sources of fatigue cracks and apply to membrane, bending, and shear stresses. Examples are:

a. Stress at the corner of a discontinuity.
b. Thermal stresses in a wall caused by a sudden change in the surface temperature.
c. Thermal stresses in cladding or weld overlay.
d. Stress due to notch effect (stress concentration).

Categories of Stress

Once the various stresses of a component are calculated, they must be combined and this final result compared to an allowable stress (see Table 1-1). The combined classes of stress due to a combination of loads acting at the same time are stress categories. Each category has assigned limits of stress based on the hazard it represents to the vessel. The following is derived basically from ASME Code, Section VIII, Division 2, simplified for application to Division 1 vessels and allowable stresses. It should be used as a guideline only because Division 1 recognizes only two categories of stress—primary membrane stress and primary bending stress. Since the calculations of most secondary (thermal and discontinuities) and peak stresses are not included in this book, these categories can be considered for reference only. In addition, Division 2 utilizes a factor K multiplied by the allowable stress for increase due to short-term loads due to seismic or upset conditions. It also sets allowable limits of combined stress for fatigue loading where secondary and peak stresses are major considerations. Table 1-1 sets allowable stresses for both stress classifications and stress categories.

Table 1-1

Allowable Stresses for Stress Classifications and Categories

Stress Classification or Category	Allowable Stress
General primary membrane, P_m	SE
General primary bending, P_b	1.5SE < $.9F_y$
Local primary membrane, P_L	
$(P_L = P_m + Q_{ms})$	1.5SE < $.9F_y$
Secondary membrane, Q_m	1.5SE < $.9F_y$
Secondary bending, Q_b	3SE < $2F_y$ < UTS
Peak, F	$2S_a$
$P_m + P_b + Q_m^* + Q_b$	3SE < $2F_y$ < UTS
$P_L + P_b$	1.5SE < $.9F_y$
$P_m + P_b + Q_m^* + Q_b$	3SE < $2F_y$ < UTS
$P_m + P_b + Q_m^* + Q_b + F$	$2S_a$

Notes:

Q_{ms} = membrane stresses from sustained loads

Q_m^* = membrane stresses from relenting, self-limiting loads

S = allowable stress per ASME Code, Section VIII, Division 1, at design temperature

F_y = minimum specified yield strength at design temperature

UTS = minimum specified tensile strength

S_a = allowable stress for any given number of cycles from design fatigue curves.

SPECIAL PROBLEMS

This book provides detailed methods to cover those areas most frequently encountered in pressure vessel design. The topics chosen for this section, while of the utmost interest to the designer, represent problems of a specialized nature. As such, they are presented here for information purposes, and detailed solutions are not provided. The solutions to these special problems are complicated and normally beyond the expertise or available time of the average designer.

The designer should be familiar with these topics in order to recognize when special consideration is warranted. If more detailed information is desired, there is a great deal of reference material available, and special references have been included for this purpose. Whenever solutions to problems in any of these areas are required, the design or analysis should be referred to experts in the field who have proven experience in their solution.

Thick-Walled Pressure Vessels

As discussed previously, the equations used for design of thin-walled vessels are inadequate for design or prediction of failure of thick-walled vessels where $R_m/t < 10$. There are many types of vessels in the thick-walled vessel category as outlined in the following, but for purposes of discussion here only the monobloc type will be discussed. Design of thick-wall vessels or cylinders is beyond the scope of this book, but it is hoped that through the following discussion some insight will be gained.

In a thick-walled vessel subjected to internal pressure, both circumferential and radial stresses are maximum on the inside surface. However, failure of the shell does not begin at the bore but in fibers along the outside surface of the shell. Although the fibers on the inside surface do reach yield first they are incapable of failing because they are restricted by the outer portions of the shell. Above the elastic-breakdown pressure the region of plastic flow or "overstrain" moves radially outward and causes the circumferential stress to reduce at the inner layers and to increase at the outer layers. Thus the maximum hoop stress is reached first at the outside of the cylinder and eventual failure begins there.

The major methods for manufacture of thick-walled pressure vessels are as follows:

1. *Monobloc*—Solid vessel wall.
2. *Multilayer*—Begins with a core about ½ in. thick and successive layers are applied. Each layer is vented (except the core) and welded individually with no overlapping welds.
3. *Multiwall*—Begins with a core about 1½ in. to 2 in. thick. Outer layers about the same thickness are successively "shrunk fit" over the core. This creates compressive stress in the core, which is relaxed during pressurization. The process of compressing layers is called autofrettage from the French word meaning "self-hooping."
4. *Multilayer autofirettage*—Begins with a core about ½ in. thick. Bands or forged rings are slipped outside

and then the core is expanded hydraulically. The core is stressed into plastic range but below ultimate strength. The outer rings are maintained at a margin below yield strength. The elastic deformation residual in the outer bands induces compressive stress in the core, which is relaxed during pressurization.

5. *Wire wrapped vessels*—Begin with inner core of thickness less than required for pressure. Core is wrapped with steel cables in tension until the desired autofrettage is achieved.
6. *Coil wrapped vessels*—Begin with a core that is subsequently wrapped or coiled with a thin steel sheet until the desired thickness is obtained. Only two longitudinal welds are used, one attaching the sheet to the core and the final closure weld. Vessels 5 to 6 ft in diameter for pressures up to 5,000 psi have been made in this manner.

Other techniques and variations of the foregoing have been used but these represent the major methods. Obviously these vessels are made for very high pressures and are very expensive.

For materials such as mild steel, which fail in shear rather than direct tension, the maximum shear theory of failure should be used. For internal pressure only, the maximum shear stress occurs on the inner surface of the cylinder. At this surface both tensile and compressive stresses are maximum. In a cylinder, the maximum tensile stress is the circumferential stress, σ_ϕ. The maximum compressive stress is the radial stress, σ_r. These stresses would be computed as follows:

$$\sigma_\phi = \frac{PR_i^2}{R_o^2 - R_i^2}\left(1 + \frac{R_o^2}{R_i^2}\right) = (+)$$

$$\sigma_r = \frac{PR_i^2}{R_o^2 - R_i^2}\left(1 - \frac{R_o^2}{R_i^2}\right) = (-)$$

Therefore the maximum shear stress, τ, is [9]:

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_\phi - \sigma_r}{2} = \frac{PR_o^2}{R_o^2 - R_i^2}$$

ASME Code, Section VIII, Division 1, has developed alternate equations for thick-walled monobloc vessels. The equations for thickness of cylindrical shells and spherical shells are as follows:

- *Cylindrical shells* (Para. 1-2 (a) (1)) where $t > .5\ R_i$ or $P > .385\ SE$:

$$Z = \frac{SE + P}{SE - P}$$

$$t = \frac{R_o(\sqrt[3]{Z} - 1)}{Z}$$

- *Spherical shells* (Para. 1-3) where $t > .356\ R_i$ or $P > .665\ SE$:

$$Y = \frac{2(SE + P)}{2SE - P}$$

$$t = R_o\left(\frac{\sqrt[3]{Y} - 1}{\sqrt[3]{Y}}\right)$$

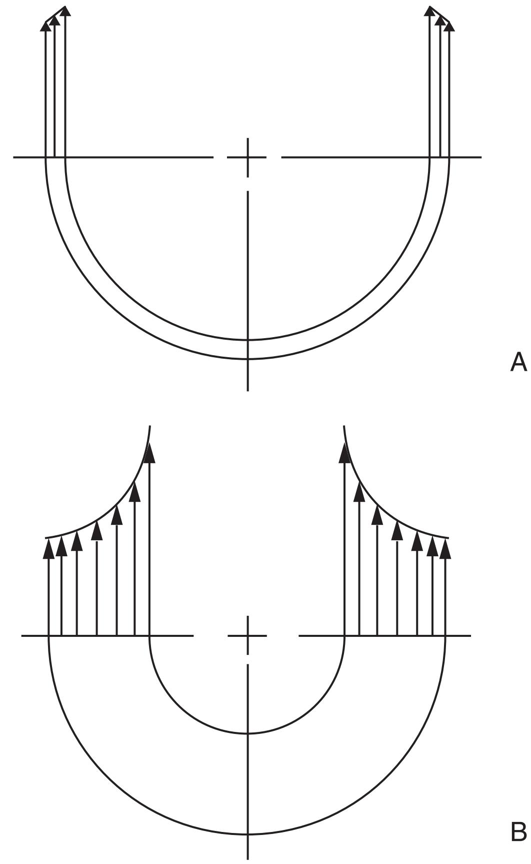

Figure 1-3. Comparision of stress distribution between thin-walled (A) and thick-walled (B) vessels.

The stress distribution in the vessel wall of a thick-walled vessel varies across the section. This is also true for thin-walled vessels, but for purposes of analysis the stress is considered uniform since the difference between the inner and outer surface is slight. A visual comparison is offered in Figure 1-3.

Thermal Stresses

Whenever the expansion or contraction that would occur normally as a result of heating or cooling an object is prevented, thermal stresses are developed. The stress is always caused by some form of mechanical restraint.

Thermal stresses are "secondary stresses" because they are self-limiting. That is, yielding or deformation of the part relaxes the stress (except thermal stress ratcheting). Thermal stresses will not cause failure by rupture in ductile materials except by fatigue over repeated applications. They can, however, cause failure due to excessive deformations.

Mechanical restraints are either internal or external. External restraint occurs when an object or component is supported or contained in a manner that restricts thermal movement. An example of external restraint occurs when piping expands into a vessel nozzle creating a radial load on the vessel shell. Internal restraint occurs when the temperature through an object is not uniform. Stresses from a "thermal gradient" are due to internal restraint. Stress is caused by a thermal gradient whenever the temperature distribution or variation within a member creates a differential expansion such that the natural growth of one fiber is influenced by the different growth requirements of adjacent fibers. The result is distortion or warpage.

A transient thermal gradient occurs during heat-up and cool-down cycles where the thermal gradient is changing with time.

Thermal gradients can be logarithmic or linear across a vessel wall. Given a steady heat input inside or outside a tube the heat distribution will be logarithmic if there is a temperature difference between the inside and outside of the tube. This effect is significant for thick-walled vessels. A linear temperature distribution occurs if the wall is thin. Stress calculations are much simpler for linear distribution.

Thermal stress ratcheting is progressive incremental inelastic deformation or strain that occurs in a component that is subjected to variations of mechanical and thermal stress. Cyclic strain accumulation ultimately can lead to incremental collapse. Thermal stress ratcheting is the result of a sustained load and a cyclically applied temperature distribution.

The fundamental difference between mechanical stresses and thermal stresses lies in the nature of the loading. Thermal stresses as previously stated are a result of restraint or temperature distribution. The fibers at high temperature are compressed and those at lower temperatures are stretched. The stress pattern must only satisfy the requirements for equilibrium of the internal forces. The result being that yielding will relax the thermal stress. If a part is loaded mechanically beyond its yield strength, the part will continue to yield until it breaks, unless the deflection is limited by strain hardening or stress redistribution. The external load remains constant, thus the internal stresses cannot relax.

The basic equations for thermal stress are simple but become increasingly complex when subjected to variables such as thermal gradients, transient thermal gradients, logarithmic gradients, and partial restraint. The basic equations follow. If the temperature of a unit cube is changed from T_1 to T_2 and the growth of the cube is fully restrained:

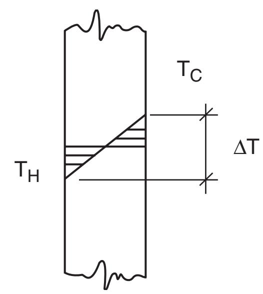

Figure 1-4. Thermal linear gradient across shell wall.

where T_1 = initial temperature, °F
T_2 = new temperature, °F
α = mean coefficient of thermal expansion in./in./°F
E = modulus of elasticity, psi
v = Poisson's ratio = .3 for steel
ΔT = mean temperature difference, °F

Case 1: If the bar is restricted only in one direction but free to expand in the other direction, the resulting uniaxial stress, σ, would be

$$\sigma = -E\alpha(T_2 - T_1)$$

- If $T_2 > T_1$, σ is compressive (expansion).
- If $T_1 > T_2$, σ is tensile (contraction).

Case 2: If restraint is in both directions, x and y, then:

$$\sigma_x = \sigma_y = -\alpha E\ \Delta T/1 - v$$

Case 3: If restraint is in all three directions, x, y, and z, then

$$\sigma_x = \sigma_y = \sigma_z = -\alpha E\ \Delta T/1 - 2v$$

Case 4: If a thermal linear gradient is across the wall of a thin shell (see Figure 1–4), then:

$$\sigma_x = \sigma_\phi = \pm\alpha E\ \Delta T/2(1 - v)$$

This is a bending stress and not a membrane stress. The hot side is in tension, the cold side in compression. Note that this is independent of vessel diameter or thickness. The stress is due to internal restraint.

Discontinuity Stresses

Vessel sections of different thickness, material, diameter, and change in directions would all have different displacements if allowed to expand freely. However, since they are connected in a continuous structure, they must deflect and rotate together. The stresses in the respective parts at or near the juncture are called discontinuity stresses. Discontinuity stresses are necessary to satisfy compatibility of deformation in the region. They are local in extent but can be of very high magnitude. Discontinuity stresses are "secondary stresses" and are self-limiting. That is, once the structure has yielded, the stresses are reduced. In average application they will not lead to failure. Discontinuity stresses do become an important factor in fatigue design where cyclic loading is a consideration. Design of the juncture of the two parts is a major consideration in reducing discontinuity stresses.

In order to find the state of stress in a pressure vessel, it is necessary to find both the membrane stresses and the discontinuity stresses. From superposition of these two states of stress, the total stresses are obtained. Generally when combined, a higher allowable stress is permitted. Due to the complexity of determining discontinuity stress, solutions will not be covered in detail here. The designer should be aware that for designs of high pressure (>1,500 psi), brittle material or cyclic loading, discontinuity stresses may be a major consideration.

Since discontinuity stresses are self-limiting, allowable stresses can be very high. One example specifically addressed by the ASME Code, Section VIII, Division 1, is discontinuity stresses at cone-cylinder intersections where the included angle is greater than 60°. Para. 1-5(e) recommends limiting combined stresses (membrane + discontinuity) in the longitudinal direction to 4SE and in the circumferential direction to 1.5SE.

ASME Code, Section VIII, Division 2, limits the combined stress, primary membrane and discontinuity stresses to $3S_m$, where S_m is the lesser of $\frac{2}{3}F_y$ or $\frac{1}{3}$U.T.S., whichever is lower.

There are two major methods for determining discontinuity stresses:

1. *Displacement Method*—Conditions of equilibrium are expressed in terms of displacement.
2. *Force Method*—Conditions of compatibility of displacements are expressed in terms of forces.

See References 2, Article 4–7; 6, Chapter 8; and 7, Chapter 4 for detailed information regarding calculation of discontinuity stresses.

Fatigue Analysis

ASME Code, Section VIII, Division 1, does not specifically provide for design of vessels in cyclic service. Although considered beyond the scope of this text as well, the designer must be aware of conditions that would require a fatigue analysis to be made.

When a vessel is subject to repeated loading that could cause failure by the development of a progressive fracture, the vessel is in cyclic service. ASME Code, Section VIII, Division 2, has established specific criteria for determining when a vessel must be designed for fatigue.

It is recognized that Code formulas for design of details, such as heads, can result in yielding in localized regions. Thus localized stresses exceeding the yield point may be encountered even though low allowable stresses have been used in the design. These vessels, while safe for relatively static conditions of loading, would develop "progressive fracture" after a large number of repeated loadings due to these high localized and secondary bending stresses. It should be noted that vessels in cyclic service require special consideration in both design *and* fabrication.

Fatigue failure can also be a result of thermal variations as well as other loadings. Fatigue failure has occurred in boiler drums due to temperature variations in the shell at the feed water inlet. In cases such as this, design details are of extreme importance.

Behavior of metal under fatigue conditions varies significantly from normal stress-strain relationships. Damage accumulates during each cycle of loading and develops at localized regions of high stress until subsequent repetitions finally cause visible cracks to grow, join, and spread. Design details play a major role in eliminating regions of stress raisers and discontinuities. It is not uncommon to have the design strength cut in half by poor design details. Progressive fractures develop from these discontinuities even though the stress is well below the static elastic strength of the material.

In fatigue service the localized stresses at abrupt changes in section, such as at a head junction or nozzle opening, misalignment, defects in construction, and thermal gradients *are* the significant stresses.

The determination of the need for a fatigue evaluation is in itself a complex job best left to those experienced in this type of analysis. For specific requirements for determining if a fatigue analysis is required see ASME Code, Section VIII, Division 2, Para. AD-160.

For additional information regarding designing pressure vessels for fatigue see Reference 7, Chapter 5.

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