CHAPTER

Introduction: Basic Principles

1

Take your choice of those that can best aid your action.
Shakespeare, *Coriolanus*

1.1 DEFINITION OF A TURBOMACHINE

We classify as turbomachines all those devices in which energy is transferred either to, or from, a continuously flowing fluid by the *dynamic action* of one or more moving blade rows. The word *turbo* or *turbinis* is of Latin origin and implies that which spins or whirls around. Essentially, a rotating blade row, a *rotor* or an *impeller* changes the stagnation enthalpy of the fluid moving through it by doing either positive or negative work, depending upon the effect required of the machine. These enthalpy changes are intimately linked with the pressure changes occurring simultaneously in the fluid.

Two main categories of turbomachine are identified: firstly, those that *absorb* power to increase the fluid pressure or head (ducted and unducted fans, compressors, and pumps); secondly, those that *produce* power by expanding fluid to a lower pressure or head (wind, hydraulic, steam, and gas turbines). Figure 1.1 shows, in a simple diagrammatic form, a selection of the many varieties of turbomachines encountered in practice. The reason that so many different types of either pump (compressor) or turbine are in use is because of the almost infinite range of service requirements. Generally speaking, for a given set of operating requirements one type of pump or turbine is best suited to provide optimum conditions of operation.

Turbomachines are further categorised according to the nature of the flow path through the passages of the rotor. When the path of the *through-flow* is wholly or mainly parallel to the axis of rotation, the device is termed an *axial flow turbomachine* [e.g., Figures 1.1(a) and (e)]. When the path of the *through-flow* is wholly or mainly in a plane perpendicular to the rotation axis, the device is termed a *radial flow turbomachine* [e.g., Figure 1.1(c)]. More detailed sketches of radial flow machines are given in Figures 7.3, 7.4, 8.2, and 8.3. *Mixed flow turbomachines* are widely used. The term *mixed flow* in this context refers to the direction of the through-flow at the rotor outlet when both radial and axial velocity components are present in significant amounts. Figure 1.1(b) shows a mixed flow pump and Figure 1.1(d) a mixed flow hydraulic turbine.

One further category should be mentioned. All turbomachines can be classified as either *impulse* or *reaction* machines according to whether pressure changes are absent or present, respectively, in the flow through the rotor. In an impulse machine all the pressure change takes place in one or more nozzles, the fluid being directed onto the rotor. The Pelton wheel, Figure 1.1(f), is an example of an impulse turbine.

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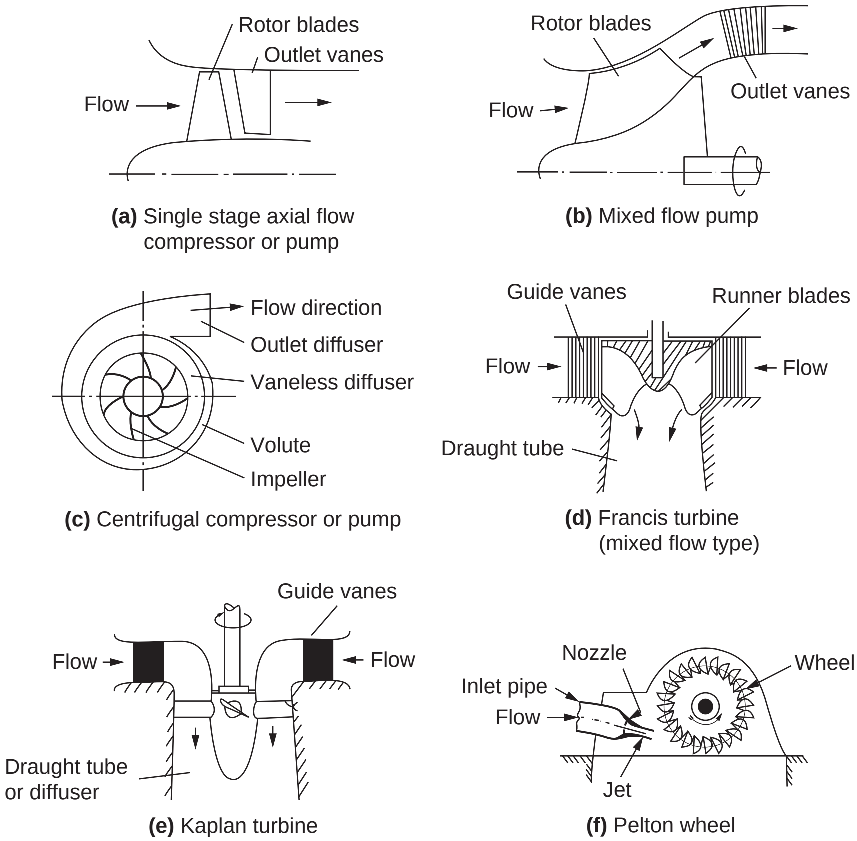

FIGURE 1.1

Examples of Turbomachines

The main purpose of this book is to examine, through the laws of fluid mechanics and thermodynamics, the means by which the energy transfer is achieved in the chief types of turbomachines, together with the differing behaviour of individual types in operation. Methods of analysing the flow processes differ depending upon the geometrical configuration of the machine, whether the fluid can be regarded as incompressible or not, and whether the machine absorbs or produces work. As far as possible, a unified treatment is adopted so that machines having similar configurations and function are considered together.

1.2 COORDINATE SYSTEM

Turbomachines consist of rotating and stationary blades arranged around a common axis, which means that they tend to have some form of cylindrical shape. It is therefore natural to use a cylindrical polar coordinate system aligned with the axis of rotation for their description and analysis. This coordinate system is pictured in Figure 1.2. The three axes are referred to as axial x, radial r, and tangential (or circumferential) $r\theta$.

In general, the flow in a turbomachine has components of velocity along all three axes, which vary in all directions. However, to simplify the analysis it is usually assumed that the flow does not vary in the tangential direction. In this case, the flow moves through the machine on *axi symmetric stream surfaces*, as drawn on Figure 1.2(a). The component of velocity along an axi-symmetric stream surface is called the *meridional* velocity,

$$c_m = \sqrt{c_x^2 + c_r^2}. \tag{1.1}$$

In purely axial-flow machines the radius of the flow path is constant and therefore, referring to Figure 1.2(c) the radial flow velocity will be zero and $c_m = c_x$. Similarly, in purely radial flow

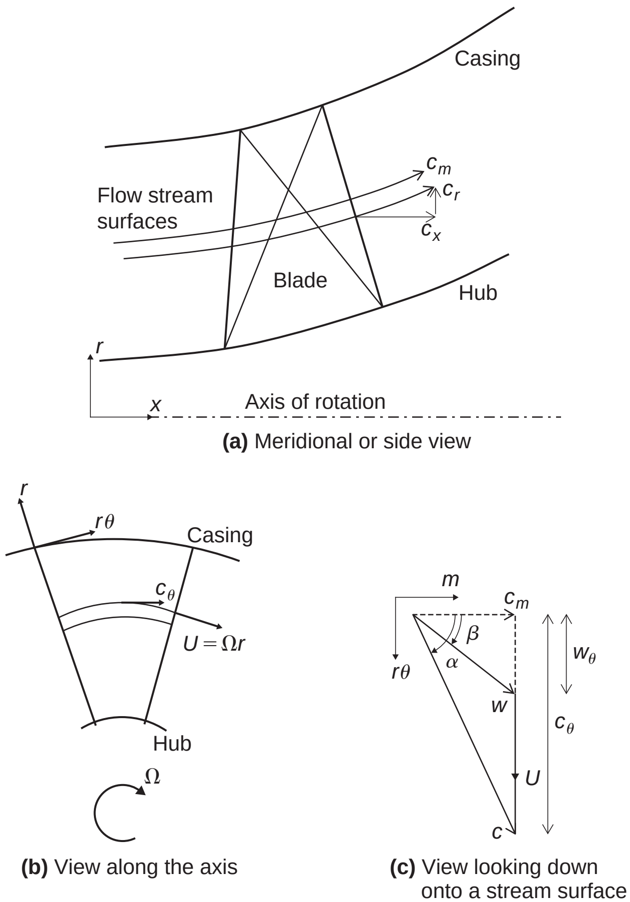

FIGURE 1.2

The Co-ordinate System and Flow Velocities within a Turbomachine machines the axial flow velocity will be zero and $c_m = c_r$. Examples of both of these types of machines can be found in Figure 1.1.

The total flow velocity is made up of the meridional and tangential components and can be written

$$c = \sqrt{c_x^2 + c_r^2 + c_\theta^2} = \sqrt{c_m^2 + c_\theta^2}. \quad (1.2)$$

The swirl, or tangential, angle is the angle between the flow direction and the meridional direction:

$$\alpha = \tan^{-1}(c_\theta / c_m). \quad (1.3)$$

Relative Velocities

The analysis of the flow-field within the rotating blades of a turbomachine is performed in a frame of reference that is stationary relative to the blades. In this frame of reference the flow appears as *steady*, whereas in the absolute frame of reference it would be *unsteady*. This makes any calculations significantly more straightforward, and therefore the use of relative velocities and relative flow quantities is fundamental to the study of turbomachinery.

The relative velocity is simply the absolute velocity minus the local velocity of the blade. The blade has velocity only in the tangential direction, and therefore the relative components of velocity can be written as

$$w_\theta = c_\theta - U, w_x = c_x, w_r = c_r. \quad (1.4)$$

The relative flow angle is the angle between the relative flow direction and the meridional direction:

$$\beta = \tan^{-1}(w_\theta / c_m). \quad (1.5)$$

By combining eqns. (1.3), (1.4), and (1.5) a relationship between the relative and absolute flow angles can be found:

$$\tan\beta = \tan\alpha - U/c_m. \quad (1.6)$$

1.3 THE FUNDAMENTAL LAWS

The remainder of this chapter summarises the basic physical laws of fluid mechanics and thermodynamics, developing them into a form suitable for the study of turbomachines. Following this, some of the more important and commonly used expressions for the efficiency of compression and expansion flow processes are given.

The laws discussed are

(i) the *continuity of flow equation*;
(ii) the *first law of thermodynamics* and the *steady flow energy equation*;
(iii) the *momentum equation*;
(iv) the *second law of thermodynamics*.

All of these laws are usually covered in first-year university engineering and technology courses, so only the briefest discussion and analysis is given here. Some textbooks dealing comprehensively

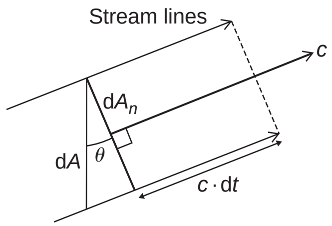

FIGURE 1.3

Flow Across an Element of Area

with these laws are those written by Çengel and Boles (1994); Douglas, Gasiorek, and Swaffield (1995); Rogers and Mayhew (1992); and Reynolds and Perkins (1977). It is worth remembering that these laws are completely general; they are independent of the nature of the fluid or whether the fluid is compressible or incompressible.

1.4 THE EQUATION OF CONTINUITY

Consider the flow of a fluid with density ρ, through the element of area dA, during the time interval dt. Referring to Figure 1.3, if c is the stream velocity the elementary mass is $\mathrm{d}m = \rho c \mathrm{d}t \mathrm{d}A \cos\theta$, where θ is the angle subtended by the normal of the area element to the stream direction. The element of area perpendicular to the flow direction is $\mathrm{d}A_n = \mathrm{d}A \cos\theta$ and so $\mathrm{d}m = \rho c \mathrm{d}A_n \mathrm{d}t$. The elementary rate of mass flow is therefore

$$\mathrm{d}\dot{m} = \frac{\mathrm{d}m}{\mathrm{d}t} = \rho c \mathrm{d}A_n. \tag{1.7}$$

Most analyses in this book are limited to one-dimensional steady flows where the velocity and density are regarded as constant across each section of a duct or passage. If A_{n1} and A_{n2} are the areas normal to the flow direction at stations 1 and 2 along a passage respectively, then

$$\dot{m} = \rho_1 c_1 \mathrm{A}_{n1} = \rho_2 c_2 A_{n2} = \rho c A_n, \tag{1.8}$$

since there is no accumulation of fluid within the control volume.

1.5 THE FIRST LAW OF THERMODYNAMICS

The *first law of thermodynamics* states that, if a system is taken through a complete cycle during which heat is supplied and work is done, then

$$\oint (\mathrm{d}Q - \mathrm{d}W) = 0, \tag{1.9}$$

where $\oint \mathrm{d}Q$ represents the heat supplied to the system during the cycle and $\oint \mathrm{d}W$ the work done by the system during the cycle. The units of heat and work in eqn. (1.9) are taken to be the same.

During a change from state 1 to state 2, there is a change in the energy within the system:

$$E_2 - E_1 = \int_1^2 (\mathrm{d}Q - \mathrm{d}W), \tag{1.10a}$$

where $E = U + \frac{1}{2}mc^2 + mgz$.

For an infinitesimal change of state,

$$\mathrm{d}E = \mathrm{d}Q - \mathrm{d}W. \tag{1.10b}$$

The Steady Flow Energy Equation

Many textbooks, e.g., Çengel and Boles (1994), demonstrate how the first law of thermodynamics is applied to the steady flow of fluid through a control volume so that the steady flow energy equation is obtained. It is unprofitable to reproduce this proof here and only the final result is quoted. Figure 1.4 shows a control volume representing a turbomachine, through which fluid passes at a steady rate of mass flow $\dot{m}$, entering at position 1 and leaving at position 2. Energy is transferred from the fluid to the blades of the turbomachine, positive work being done (via the shaft) at the rate $\dot{W}_x$. In the general case positive heat transfer takes place at the rate $\dot{Q}$, *from* the surroundings *to* the control volume. Thus, with this sign convention the steady flow energy equation is

$$\dot{Q} - \dot{W}_x = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1)\right], \tag{1.11}$$

where h is the specific enthalpy, $\frac{1}{2}c^2$, the kinetic energy per unit mass and gz, the potential energy per unit mass.

For convenience, the specific enthalpy, h, and the kinetic energy, $\frac{1}{2}c^2$, are combined and the result is called the *stagnation enthalpy*:

$$h_0 = h + \frac{1}{2}c^2. \tag{1.12}$$

Apart from hydraulic machines, the contribution of the $g(z_2 - z_1)$ term in eqn. (1.11) is small and can usually ignored. In this case, eqn. (1.11) can be written as

$$\dot{Q} - \dot{W}_x = \dot{m}(h_{02} - h_{01}). \tag{1.13}$$

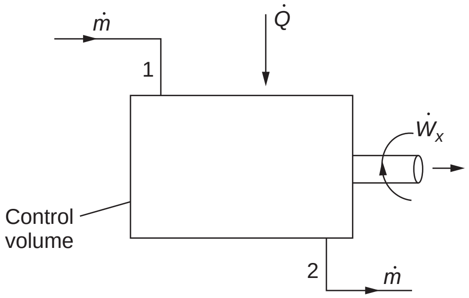

FIGURE 1.4

Control Volume Showing Sign Convention for Heat and Work Transfers

The stagnation enthalpy is therefore constant in any flow process that does not involve a work transfer or a heat transfer. Most turbomachinery flow processes are adiabatic (or very nearly so) and it is permissible to write $\dot{Q} = 0$. For work producing machines (turbines) $\dot{W}_x > 0$, so that

$$\dot{W}_x = \dot{W}_t = \dot{m}(h_{01} - h_{02}). \tag{1.14}$$

For work absorbing machines (compressors) $\dot{W}_x < 0$, so that it is more convenient to write

$$\dot{W}_c = -\dot{W}_x = \dot{m}(h_{02} - h_{01}). \tag{1.15}$$

1.6 THE MOMENTUM EQUATION

One of the most fundamental and valuable principles in mechanics is *Newton's second law of motion*. The momentum equation relates the sum of the external forces acting on a fluid element to its acceleration, or to the rate of change of momentum in the direction of the resultant external force. In the study of turbomachines many applications of the momentum equation can be found, e.g., the force exerted upon a blade in a compressor or turbine cascade caused by the deflection or acceleration of fluid passing the blades.

Considering a system of mass m, the sum of all the body and surface forces acting on m along some arbitrary direction x is equal to *the time rate of change of the total x-momentum of the system*, i.e.,

$$\sum F_x = \frac{\mathrm{d}}{\mathrm{d}t}(mc_x). \tag{1.16a}$$

For a control volume where fluid enters steadily at a uniform velocity c_{x1} and leaves steadily with a uniform velocity c_{x2}, then

$$\sum F_x = \dot{m}(c_{x2} - c_{x1}). \tag{1.16b}$$

Equation (1.16b) is the one-dimensional form of the steady flow momentum equation.

Moment of Momentum

In dynamics useful information can be obtained by employing Newton's second law in the form where it applies to the moments of forces. This form is of central importance in the analysis of the energy transfer process in turbomachines.

For a system of mass m, the vector sum of the moments of all external forces acting on the system about some arbitrary axis A–A fixed in space is equal to the time rate of change of angular momentum of the system about that axis, i.e.,

$$\tau_A = m\frac{\mathrm{d}}{\mathrm{d}t}(rc_\theta), \tag{1.17a}$$

where r is distance of the mass centre from the axis of rotation measured along the normal to the axis and c_θ the velocity component mutually perpendicular to both the axis and radius vector r.

For a control volume the *law of moment of momentum* can be obtained. Figure 1.5 shows the control volume enclosing the rotor of a generalised turbomachine. Swirling fluid enters the control volume

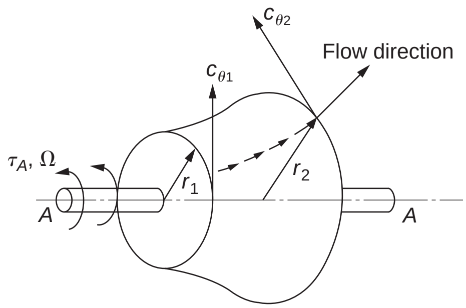

FIGURE 1.5

Control Volume for a Generalised Turbomachine

at radius r_1 with tangential velocity $c_{\theta 1}$ and leaves at radius r_2 with tangential velocity $c_{\theta 2}$. For one-dimensional steady flow,

$$\tau_A = \dot{m}(r_2 c_{\theta 2} - r_1 c_{\theta 1}), \tag{1.17b}$$

which states that the sum of the moments of the external forces acting on fluid temporarily occupying the control volume is equal to the net time rate of efflux of angular momentum from the control volume.

The Euler Work Equation

For a pump or compressor rotor running at angular velocity Ω, the rate at which the rotor does work on the fluid is

$$\tau_A \Omega = \dot{m}(U_2 c_{\theta 2} - U_1 c_{\theta 1}), \tag{1.18a}$$

where the blade speed $U = \Omega r$.

Thus, the work done on the fluid per unit mass or specific work is

$$\Delta W_c = \frac{\dot{W}_c}{\dot{m}} = \frac{\tau_A \Omega}{\dot{m}} = U_2 c_{\theta 2} - U_1 c_{\theta 1} > 0. \tag{1.18b}$$

This equation is referred to as *Euler's pump equation*.

For a turbine the fluid does work *on* the rotor and the sign for work is then reversed. Thus, the specific work is

$$\Delta W_t = \frac{\dot{W}_t}{\dot{m}} = U_1 c_{\theta 1} - U_2 c_{\theta 2} > 0. \tag{1.18c}$$

Equation (1.18c) is referred to as *Euler's turbine equation*.

Note that, for any adiabatic turbomachine (turbine or compressor), applying the steady flow energy equation, eqn. (1.13), gives

$$\Delta W_x = (h_{01} - h_{02}) = U_1 c_{\theta 1} - U_2 c_{\theta 2}. \tag{1.19a}$$

Alternatively, this can be written as

$$\Delta h_0 = \Delta(U c_\theta). \tag{1.19b}$$

Equations (1.19a) and (1.19b) are the general forms of the *Euler work equation*. By considering the assumptions used in its derivation, this equation can be seen to be valid for adiabatic flow for any streamline through the blade rows of a turbomachine. It is applicable to both viscous and inviscid flow, since the torque provided by the fluid on the blades can be exerted by pressure forces or frictional forces. It is strictly valid only for steady flow but it can also be applied to time-averaged unsteady flow provided the averaging is done over a long enough time period. In all cases, all of the torque from the fluid must be transferred to the blades. Friction on the hub and casing of a turbomachine can cause local changes in angular momentum that are not accounted for in the Euler work equation.

Note that for any stationary blade row, $U = 0$ and therefore $h_0 =$ constant. This is to be expected since a stationary blade cannot transfer any work to or from the fluid.

Rothalpy and Relative Velocities

The Euler work equation, eqn. (1.19), can be rewritten as

$$I = h_0 - Uc_\theta, \tag{1.20a}$$

where I is a constant along the streamlines through a turbomachine. The function I has acquired the widely used name *rothalpy*, a contraction of rotational stagnation enthalpy, and is a fluid mechanical property of some importance in the study of flow within rotating systems. The rothalpy can also be written in terms of the static enthalpy as

$$I = h + \frac{1}{2}c^2 - Uc_\theta. \tag{1.20b}$$

The Euler work equation can also be written in terms of relative quantities for a rotating frame of reference. The relative tangential velocity, as given in eqn. (1.4), can be substituted in eqn. (1.20b) to produce

$$I = h + \frac{1}{2}(w^2 + U^2 + 2Uw_\theta) - U(w_\theta + U) = h + \frac{1}{2}w^2 - \frac{1}{2}U^2. \tag{1.21a}$$

Defining a relative stagnation enthalpy as $h_{0,\mathrm{rel}} = h + \frac{1}{2}w^2$, eqn. (1.21a) can be simplified to

$$I = h_{0,\mathrm{rel}} - \frac{1}{2}U^2. \tag{1.21b}$$

This final form of the Euler work equation shows that, for rotating blade rows, the relative stagnation enthalpy is constant through the blades provided the blade speed is constant. In other words, $h_{0,\mathrm{rel}} =$ constant, if the radius of a streamline passing through the blades stays the same. This result is important for analysing turbomachinery flows in the relative frame of reference.

1.7 THE SECOND LAW OF THERMODYNAMICS—ENTROPY

The *second law of thermodynamics*, developed rigorously in many modern thermodynamic textbooks, e.g., Çengel and Boles (1994), Reynolds and Perkins (1977), and Rogers and Mayhew (1992), enables the concept of entropy to be introduced and ideal thermodynamic processes to be defined.

An important and useful corollary of the second law of thermodynamics, known as the *inequality of Clausius*, states that, for a system passing through a cycle involving heat exchanges,

$$\oint \frac{\mathrm{d}Q}{T} \leq 0, \tag{1.22a}$$

where $\mathrm{d}Q$ is an element of heat transferred to the system at an absolute temperature T. If all the processes in the cycle are reversible, then $\mathrm{d}Q = \mathrm{d}Q_R$, and the equality in eqn. (1.22a) holds true, i.e.,

$$\oint \frac{\mathrm{d}Q_R}{T} = 0. \tag{1.22b}$$

The property called *entropy*, for a finite change of state, is then defined as

$$S_2 - S_1 = \int_1^2 \frac{\mathrm{d}Q_R}{T}. \tag{1.23a}$$

For an incremental change of state

$$\mathrm{d}S = m\mathrm{d}s = \frac{\mathrm{d}Q_R}{T}, \tag{1.23b}$$

where m is the mass of the system.

With steady one-dimensional flow through a control volume in which the fluid experiences a change of state from condition 1 at entry to 2 at exit,

$$\int_1^2 \frac{\mathrm{d}\dot{Q}}{T} \leq \dot{m}(s_2 - s_1). \tag{1.24a}$$

Alternatively, this can be written in terms of an entropy production due to irreversibility, ΔS_{irrev}:

$$\dot{m}(s_2 - s_1) = \int_1^2 \frac{\mathrm{d}\dot{Q}}{T} + \Delta S_{\text{irrev}}. \tag{1.24b}$$

If the process is adiabatic, $\mathrm{d}\dot{Q} = 0$, then

$$s_2 \geq s_1. \tag{1.25}$$

If the process is *reversible* as well, then

$$s_2 = s_1. \tag{1.26}$$

Thus, for a flow undergoing a process that is both adiabatic and reversible, the entropy will remain unchanged (this type of process is referred to as *isentropic*). Since turbomachinery is usually adiabatic, or close to adiabatic, an isentropic compression or expansion represents the best possible process that can be achieved. To maximize the efficiency of a turbomachine, the irreversible entropy production ΔS_{irrev} must be minimized, and this is a primary objective of any design.

Several important expressions can be obtained using the preceding definition of *entropy*. For a system of mass m undergoing a reversible process $\mathrm{d}Q = \mathrm{d}Q_R = mT\mathrm{d}s$ and $\mathrm{d}W = \mathrm{d}W_R = mp\mathrm{d}v$. In the absence of motion, gravity, and other effects the first law of thermodynamics, eqn. (1.10b) becomes

$$T\mathrm{d}s = \mathrm{d}u + p\mathrm{d}v. \tag{1.27}$$

With $h = u + pv$, then $dh = du + pdv + vdp$, and eqn. (1.27) then gives

$$Tds = dh - vdp. \tag{1.28}$$

Equations (1.27) and (1.28) are extremely useful forms of the second law of thermodynamics because the equations are written only in terms of properties of the system (there are no terms involving Q or W). These equations can therefore be applied to a system undergoing any process.

Entropy is a particularly useful property for the analysis of turbomachinery. Any creation of entropy in the flow path of a machine can be equated to a certain amount of "lost work" and thus a loss in efficiency. The value of entropy is the same in both the absolute and relative frames of reference (see Figure 1.7 later) and this means it can be used to track the sources of inefficiency through all the rotating and stationary parts of a machine. The application of entropy to account for lost performance is very powerful and will be demonstrated in later sections.

1.8 BERNOULLI'S EQUATION

Consider the steady flow energy equation, eqn. (1.11). For adiabatic flow, with no work transfer,

$$(h_2 - h_1) + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1) = 0. \tag{1.29}$$

If this is applied to a control volume whose thickness is infinitesimal in the stream direction (Figure 1.6), the following differential form is derived:

$$dh + cdc + gdz = 0. \tag{1.30}$$

If there are no shear forces acting on the flow (no mixing or friction), then the flow will be isentropic and, from eqn. (1.28), $dh = vdp = dp/\rho$, giving

$$\frac{1}{\rho}dp + cdc + gdz = 0. \tag{1.31a}$$

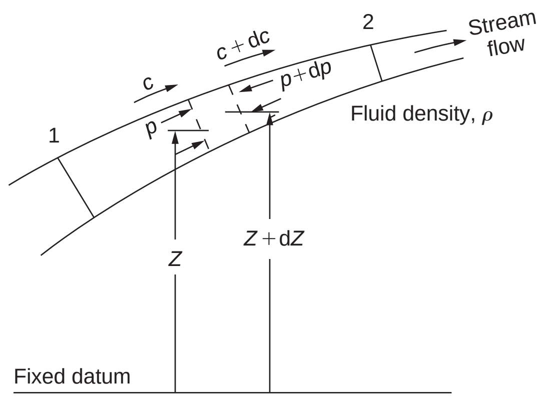

FIGURE 1.6

Control Volume in a Streaming Fluid

Equation (1.31) is often referred to as the *one-dimensional form* of Euler's equation of motion. Integrating this equation in the stream direction we obtain

$$\int_1^2 \frac{1}{\rho}\mathrm{d}p + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1) = 0, \tag{1.31b}$$

which is Bernoulli's equation. For an incompressible fluid, ρ is constant and eqn. (1.31b) becomes

$$\frac{1}{\rho}(p_{02} - p_{01}) + g(z_2 - z_1) = 0, \tag{1.31c}$$

where the stagnation pressure for an incompressible fluid is $p_0 = p + \frac{1}{2}\rho c^2$.

When dealing with hydraulic turbomachines, the term *head*, H, occurs frequently and describes the quantity $z + p_0/(\rho g)$. Thus, eqn. (1.31c) becomes

$$H_2 - H_1 = 0. \tag{1.31d}$$

If the fluid is a gas or vapour, the change in gravitational potential is generally negligible and eqn. (1.31b) is then

$$\int_1^2 \frac{1}{\rho}\mathrm{d}p + \frac{1}{2}(c_2^2 - c_1^2) = 0. \tag{1.31e}$$

Now, if the gas or vapour is subject to only small pressure changes the fluid density is sensibly constant and integration of eqn. (1.31e) gives

$$p_{02} = p_{01} = p_0, \tag{1.31f}$$

i.e., the stagnation pressure is constant (it is shown later that this is also true for a *compressible isentropic process*).

1.9 COMPRESSIBLE FLOW RELATIONS

The Mach number of a flow is defined as the velocity divided by the local speed of sound. For a perfect gas, such as air, the Mach number can be written as

$$\mathrm{M} = \frac{c}{a} = \frac{c}{\sqrt{\gamma RT}}. \tag{1.32}$$

Whenever the Mach number in a flow exceeds about 0.3, the flow becomes compressible, and the fluid density can no longer be considered as constant. High power turbomachines require high flow rates and high blade speeds and this inevitably leads to compressible flow. The static and stagnation quantities in the flow can be related using functions of the local Mach number and these are derived later.

Starting with the definition of stagnation enthalpy, $h_0 = h + \frac{1}{2}c^2$, this can be rewritten for a perfect gas as

$$C_pT_0 = C_pT + \frac{c^2}{2} = C_pT + \frac{\mathrm{M}^2\gamma RT}{2}. \tag{1.33a}$$

Given that $\gamma R = (\gamma - 1)C_P$, eqn. (1.33a) can be simplified to

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}\mathrm{M}^2. \tag{1.33b}$$

The stagnation pressure in a flow is the static pressure that is measured if the flow is brought *isentropically* to rest. From eqn. (1.28), for an isentropic process $\mathrm{d}h = \mathrm{d}p/\rho$. If this is combined with the equation of state for a perfect gas, $p = \rho RT$, the following equation is obtained:

$$\frac{\mathrm{d}p}{p} = \frac{C_p}{R}\frac{\mathrm{d}T}{T} = \frac{\mathrm{d}T}{T}\frac{\gamma}{\gamma - 1} \tag{1.34}$$

This can be integrated between the static and stagnation conditions to give the following compressible flow relation between the stagnation and static pressure:

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)} = \left(1 + \frac{\gamma - 1}{2}\mathrm{M}^2\right)^{\gamma/(\gamma-1)}. \tag{1.35}$$

Equation (1.34) can also be integrated along a streamline between any two arbitrary points 1 and 2 within an isentropic flow. In this case, the stagnation temperatures and pressures are related:

$$\frac{p_{02}}{p_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\gamma/(\gamma-1)}. \tag{1.36}$$

If there is no heat or work transfer to the flow, $T_0 =$ constant. Hence, eqn. (1.36) shows that, in isentropic flow with no work transfer, $p_{02} = p_{01} =$ constant, which was shown to be the case for incompressible flow in eqn. (1.31f).

Combining the equation of state, $p = \rho RT$ with eqns. (1.33b) and (1.35) the corresponding relationship for the stagnation density is obtained:

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2}\mathrm{M}^2\right)^{1/(\gamma-1)}. \tag{1.37}$$

Arguably the most important compressible flow relationship for turbomachinery is the one for non-dimensional mass flow rate, sometimes referred to as *capacity*. It is obtained by combining eqns. (1.33b), (1.35), and (1.37) with continuity, eqn. (1.8):

$$\frac{\dot{m}\sqrt{C_P T_0}}{A_n p_0} = \frac{\gamma}{\sqrt{\gamma - 1}}\mathrm{M}\left(1 + \frac{\gamma - 1}{2}\mathrm{M}^2\right)^{-\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}. \tag{1.38}$$

This result is important since it can be used to relate the flow properties at different points within a compressible flow turbomachine. The application of eqn. (1.38) is demonstrated in Chapter 3.

Note that the compressible flow relations given previously can be applied in the relative frame of reference for flow within rotating blade rows. In this case relative stagnation properties and relative Mach numbers are used:

$$\frac{p_{0,\mathrm{rel}}}{p}, \frac{T_{0,\mathrm{rel}}}{T}, \frac{\rho_{0,\mathrm{rel}}}{\rho}, \frac{\dot{m}\sqrt{C_p T_{0,\mathrm{rel}}}}{A p_{0,\mathrm{rel}}} = f(\mathrm{M}_{\mathrm{rel}}). \tag{1.39}$$

Figure 1.7 shows the relationship between stagnation and static conditions on a temperature–entropy diagram, in which the temperature differences have been exaggerated for clarity. This shows the relative stagnation properties as well as the absolute properties for a single point in a flow. Note that all of the conditions have the same entropy because the stagnation states are defined using an isentropic process. The pressures and temperatures are related using eqn. (1.35).

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Variation of Gas Properties with Temperature

The thermodynamic properties of a gas, C_p and γ, are dependent upon its temperature level, and some account must be taken of this effect. To illustrate this dependency the variation in the values of C_p and γ with the temperature for air are shown in Figure 1.8. In the calculation of expansion or compression processes in turbomachines the normal practice is to use weighted mean values for C_p and γ according to the mean temperature of the process. Accordingly, in all problems in this book values have been selected for C_p and γ appropriate to the gas and the temperature range.

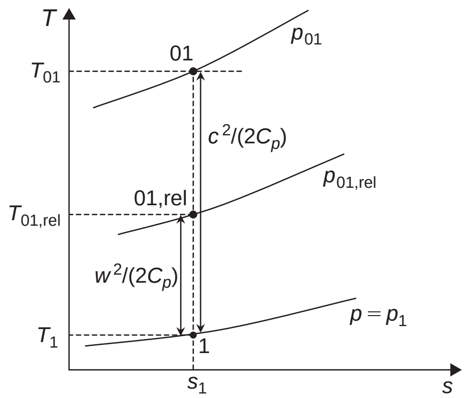


FIGURE 1.7

Relationship Between Stagnation and Static Quantities on a Temperature–Entropy Diagram

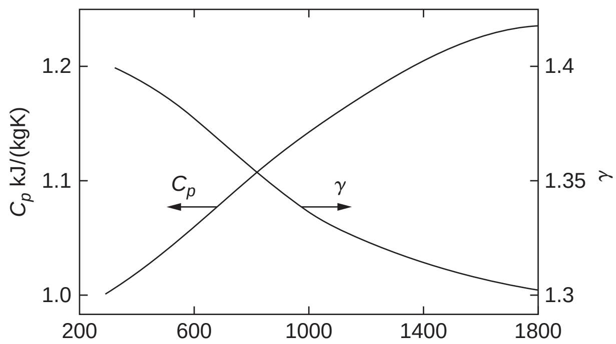


FIGURE 1.8

Variation of Gas Properties with Temperature for Dry Air (data from Rogers and Mayhew, 1995)

1.10 DEFINITIONS OF EFFICIENCY

A large number of efficiency definitions are included in the literature of turbomachines and most workers in this field would agree there are too many. In this book only those considered to be important and useful are included.

Efficiency of Turbines

Turbines are designed to convert the available energy in a flowing fluid into useful mechanical work delivered at the coupling of the output shaft. The efficiency of this process, the *overall efficiency* η_0, is a performance factor of considerable interest to both designer and user of the turbine. Thus,

$$\eta_0 = \frac{\text{mechanical energy available at coupling of output shaft in unit time}}{\text{maximum energy difference possible for the fluid in unit time}}.$$

Mechanical energy losses occur between the turbine rotor and the output shaft coupling as a result of the work done against friction at the bearings, glands, etc. The magnitude of this loss as a fraction of the total energy transferred to the rotor is difficult to estimate as it varies with the size and individual design of turbomachine. For small machines (several kilowatts) it may amount to 5% or more, but for medium and large machines this loss ratio may become as little as 1%. A detailed consideration of the mechanical losses in turbomachines is beyond the scope of this book and is not pursued further.

The *isentropic efficiency* η_t or *hydraulic efficiency* η_h for a turbine is, in broad terms,

$$\eta_t\,(\text{or}\,\eta_h) = \frac{\text{mechanical energy supplied to the rotor in unit time}}{\text{maximum energy difference possible for the fluid in unit time}}.$$

Comparing these definitions it is easily deduced that the *mechanical efficiency* η_m, which is simply the ratio of shaft power to rotor power, is

$$\eta_m = \eta_0/\eta_t\,(\text{or}\,\eta_0/\eta_h). \tag{1.40}$$

The preceding isentropic efficiency definition can be concisely expressed in terms of the work done by the fluid passing through the turbine:

$$\eta_t\,(\text{or}\,\eta_h) = \frac{\text{actual work}}{\text{ideal (maximum) work}} = \frac{\Delta W_x}{\Delta W_{\max}}. \tag{1.41}$$

The *actual work* is unambiguous and straightforward to determine from the steady flow energy equation, eqn. (1.11). For an adiabatic turbine, using the definition of stagnation enthalpy,

$$\Delta W_x = \dot{W}_x/\dot{m} = (h_{01} - h_{02}) + g(z_1 - z_2).$$

The *ideal work* is slightly more complicated as it depends on how the ideal process is defined. The process that gives maximum work will always be an isentropic expansion, but the question is one of how to define the exit state of the ideal process relative to the actual process. In the following paragraphs the different definitions are discussed in terms of to what type of turbine they are applied.

Steam and Gas Turbines

Figure 1.9(a) shows a Mollier diagram representing the expansion process through an adiabatic turbine. Line 1–2 represents the actual expansion and line 1–2*s* the ideal or reversible expansion. The fluid velocities at entry to and exit from a turbine may be quite high and the corresponding kinetic energies significant. On the other hand, for a compressible fluid the potential energy terms are usually negligible. Hence, the *actual* turbine rotor *specific work* is

$$\Delta W_x = \dot{W}_x/\dot{m} = h_{01} - h_{02} = (h_1 - h_2) + \frac{1}{2}(c_1^2 - c_2^2).$$

There are two main ways of expressing the isentropic efficiency, the choice of definition depending largely upon whether the *exit kinetic energy* is usefully employed or is wasted. If the exhaust kinetic energy is useful, then the ideal expansion is to the same stagnation (or total) pressure as the actual process. The ideal work output is therefore that obtained between state points 01 and 02*s*,

$$\Delta W_{\max} = \dot{W}_{\max}/\dot{m} = h_{01} - h_{02s} = (h_1 - h_{2s}) + \frac{1}{2}(c_1^2 - c_{2s}^2).$$

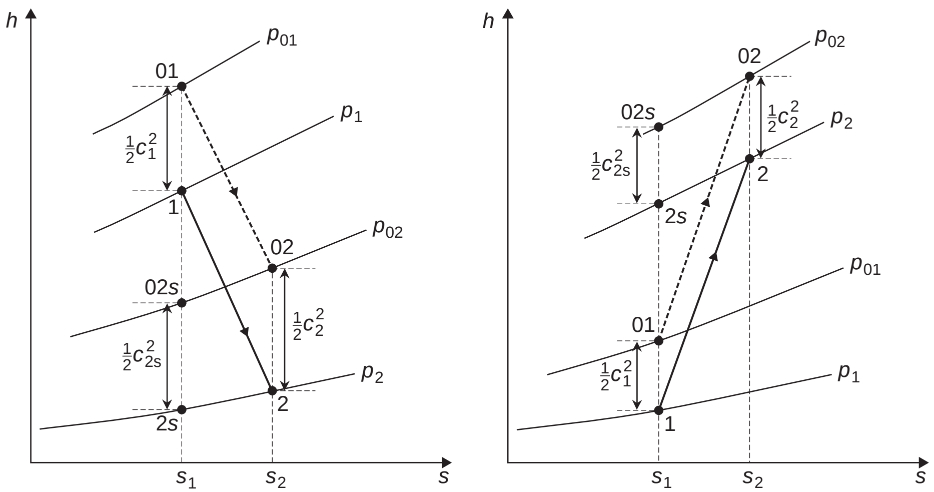
The relevant adiabatic efficiency, η, is called the *total-to-total efficiency* and it is given by

$$\eta_{tt} = \Delta W_x/\Delta W_{\max} = (h_{01} - h_{02})/(h_{01} - h_{02s}). \tag{1.42a}$$

If the difference between the inlet and outlet kinetic energies is small, i.e., $\frac{1}{2}c_1^2 \cong \frac{1}{2}c_2^2$, then

$$\eta_{tt} = (h_1 - h_2)/(h_1 - h_{2s}). \tag{1.42b}$$

An example where the exhaust kinetic energy is not wasted is from the last stage of an aircraft gas turbine where it contributes to the jet propulsive thrust. Likewise, the exit kinetic energy from one stage of a multistage turbine where it can be used in the following stage provides another example.



(a) Turbine expansion process **(b)** Compression process

FIGURE 1.9

Enthalpy–Entropy Diagrams for the Flow Through an Adiabatic Turbine and an Adiabatic Compressor

If, instead, the exhaust kinetic energy cannot be usefully employed and is entirely wasted, the ideal expansion is to the same static pressure as the actual process with zero exit kinetic energy. The ideal work output in this case is that obtained between state points 01 and 2*s*:

$$\Delta W_{\max} = \dot{W}_{\max}/\dot{m} = h_{01} - h_{2s} = (h_1 - h_{2s}) + \frac{1}{2}c_1^2.$$

The relevant adiabatic efficiency is called the *total-to-static efficiency* η_{ts} and is given by

$$\eta_{ts} = \Delta W_x/\Delta W_{\max} = (h_{01} - h_{02})/(h_{01} - h_{2s}). \tag{1.43a}$$

If the difference between inlet and outlet kinetic energies is small, eqn. (1.43a) becomes

$$\eta_{ts} = (h_1 - h_2)\Big/\left(h_1 - h_{2s} + \frac{1}{2}c_1^2\right). \tag{1.43b}$$

A situation where the outlet kinetic energy is wasted is a turbine exhausting directly to the surroundings rather than through a diffuser. For example, auxiliary turbines used in rockets often have no exhaust diffusers because the disadvantages of increased mass and space utilisation are greater than the extra propellant required as a result of reduced turbine efficiency.

By comparing eqns. (1.42) and (1.43) it is clear that the total-to-static efficiency will always be lower than the total-to-total efficiency. The total-to-total efficiency relates to the internal losses (entropy creation) within the turbine, whereas the total-to-static efficiency relates to the internal losses plus the wasted kinetic energy.

Hydraulic Turbines

The turbine hydraulic efficiency is a form of the total-to-total efficiency expressed previously. The steady flow energy equation (eqn. 1.11) can be written in differential form for an adiabatic turbine as

$$\mathrm{d}\dot{W}_x = \dot{m}\left[\mathrm{d}h + \frac{1}{2}\mathrm{d}(c^2) + g\mathrm{d}z\right].$$

For an isentropic process, $T\mathrm{d}s = 0 = \mathrm{d}h - \mathrm{d}p/\rho$. The maximum work output for an expansion to the same exit static pressure, kinetic energy, and height as the actual process is therefore

$$\dot{W}_{\max} = \dot{m}\left[\int_1^2 \frac{1}{\rho}\mathrm{d}p + \frac{1}{2}(c_1^2 - c_2^2) + g(z_1 - z_2)\right].$$

For an incompressible fluid, the maximum work output from a hydraulic turbine (ignoring frictional losses) can be written

$$\dot{W}_{\max} = \dot{m}\left[\frac{1}{\rho}(p_1 - p_2) + \frac{1}{2}(c_1^2 - c_2^2) + g(z_1 - z_2)\right] = \dot{m}g(H_1 - H_2),$$

where $gH = p/\rho + \frac{1}{2}c^2 + gz$ and $\dot{m} = \rho Q$.

The turbine hydraulic efficiency, η_h, is the work supplied by the rotor divided by the hydrodynamic energy difference of the fluid, i.e.,

$$\eta_h = \frac{\dot{W}_x}{\dot{W}_{\max}} = \frac{\Delta W_x}{g[H_1 - H_2]}. \tag{1.44}$$

Efficiency of Compressors and Pumps

The isentropic *efficiency*, η_c, of a compressor or the *hydraulic efficiency* of a pump, η_h, is broadly defined as

$$\eta_c\,(\text{or}\,\eta_h) = \frac{\text{useful (hydrodynamic) energy input to fluid in unit time}}{\text{power input to rotor}}$$

The power input to the rotor (or impeller) is always less than the power supplied at the coupling because of external energy losses in the bearings, glands, etc. Thus, the overall efficiency of the compressor or pump is

$$\eta_o = \frac{\text{useful (hydrodynamic) energy input to fluid in unit time}}{\text{power input to coupling of shaft}}.$$

Hence, the mechanical efficiency is

$$\eta_m = \eta_o/\eta_c\,(\text{or}\,\eta_o/\eta_h). \tag{1.45}$$

For a complete adiabatic compression process going from state 1 to state 2, the specific work input is

$$\Delta W_c = (h_{02} - h_{01}) + g(z_2 - z_1).$$

Figure 1.9(b) shows a Mollier diagram on which the actual compression process is represented by the state change 1–2 and the corresponding ideal process by 1–2*s*. For an adiabatic compressor in which potential energy changes are negligible, the most meaningful efficiency is the total-to-total efficiency, which can be written as

$$\eta_c = \frac{\text{ideal (minimum) work input}}{\text{actual work input}} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}}. \tag{1.46a}$$

If the difference between inlet and outlet kinetic energies is small, $\frac{1}{2}c_1^2 \cong \frac{1}{2}c_2^2$ then

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}. \tag{1.46b}$$

For *incompressible* flow, the minimum work input is given by

$$\Delta W_{\min} = \dot{W}_{\min}/\dot{m} = \left[(p_2 - p_1)/\rho + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1)\right] = g[H_2 - H_1].$$

For a pump the hydraulic efficiency is therefore defined as

$$\eta_h = \frac{\dot{W}_{\min}}{\dot{W}_c} = \frac{g[H_2 - H_1]}{\Delta W_c}. \tag{1.47}$$

1.11 SMALL STAGE OR POLYTROPIC EFFICIENCY

The isentropic *efficiency* described in the preceding section, although fundamentally valid, can be misleading if used for comparing the efficiencies of turbomachines of differing pressure ratios. Now, any turbomachine may be regarded as being composed of a large number of very small stages, irrespective of the actual number of stages in the machine. If each small stage has the same efficiency, then the isentropic efficiency of the whole machine will be different from the small stage efficiency, the difference depending upon the pressure ratio of the machine. This perhaps rather surprising result is a manifestation of a simple thermodynamic effect concealed in the expression for isentropic efficiency and is made apparent in the following argument.

Compression Process

Figure 1.10 shows an enthalpy–entropy diagram on which adiabatic compression between pressures p_1 and p_2 is represented by the change of state between points 1 and 2. The corresponding reversible process is represented by the isentropic line 1 to 2*s*. It is assumed that the compression process may be divided into a large number of small stages of equal efficiency η_p. For each small stage the actual work input is δW and the corresponding ideal work in the isentropic process is $\delta W_{\min}$. With the notation of Figure 1.10,

$$\eta_p = \frac{\delta W_{\min}}{\delta W} = \frac{h_{xs} - h_1}{h_x - h_1} = \frac{h_{ys} - h_x}{h_y - h_x} = \cdots$$

Since each small stage has the same efficiency, then $\eta_p = (\Sigma\delta W_{\min}/\Sigma\delta W)$ is also true.

From the relation $T\mathrm{d}s = \mathrm{d}h - v\mathrm{d}p$, for a constant pressure process, $(\partial h/\partial s)_{p1} = T$. This means that the higher the fluid temperature, the *greater* is the slope of the constant pressure lines on the Mollier diagram. For a gas where h is a function of T, constant pressure lines diverge and the slope of the line

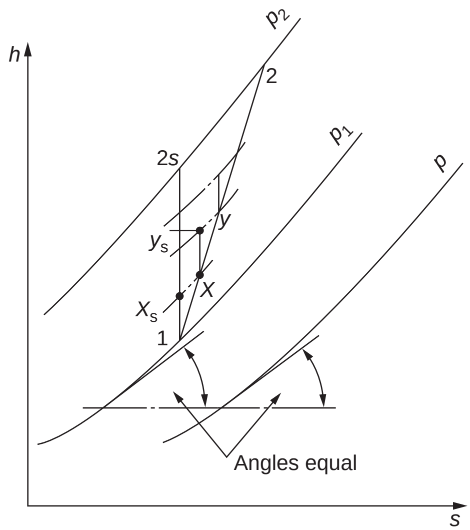


FIGURE 1.10

Compression Process by Small Stages

p_2 is greater than the slope of line p_1 at the same value of entropy. At equal values of T, constant pressure lines are of equal slope as indicated in Figure 1.10. For the special case of a *perfect gas* (where C_p is constant), $C_p(\mathrm{d}T/\mathrm{d}s) = T$ for a constant pressure process. Integrating this expression results in the equation for a constant pressure line, $s = C_p \log T + \text{constant}$.

Returning now to the more general case, since

$$\Sigma dW = \{(h_x - h_1) + (h_y - h_x) + \cdots\} = (h_2 - h_1),$$

then

$$\eta_p = \left[(h_{xs} - h_1) + (h_{ys} - h_x) + \cdots\right]/(h_2 - h_1).$$

The adiabatic efficiency of the *whole* compression process is

$$\eta_c = (h_{2s} - h_1)/(h_2 - h_1).$$

Due to the divergence of the constant pressure lines

$$\{(h_{xs} - h_1) + (h_{ys} - h_x) + \cdots\} > (h_{2s} - h_1),$$

i.e.,

$$\Sigma \delta W_{\min} > W_{\min}.$$

Therefore,

$$\eta_p > \eta_c.$$

Thus, for a compression process the isentropic efficiency of the machine is *less* than the small stage efficiency, the difference being dependent upon the divergence of the constant pressure lines. Although the foregoing discussion has been in terms of static states it also applies to stagnation states since these are related to the static states via isentropic processes.

Small Stage Efficiency for a Perfect Gas

An explicit relation can be readily derived for a perfect gas (C_p is constant) between small stage efficiency, the overall isentropic efficiency and the pressure ratio. The analysis is for the limiting case of an infinitesimal compressor stage in which the incremental change in pressure is dp as indicated in Figure 1.11. For the actual process the incremental enthalpy rise is dh and the corresponding ideal enthalpy rise is dh_{is}.

The polytropic efficiency for the small stage is

$$\eta_p = \frac{\mathrm{d}h_{is}}{\mathrm{d}h} = \frac{v\mathrm{d}p}{C_p\mathrm{d}T}, \tag{1.48}$$

since for an isentropic process $T\mathrm{d}s = 0 = \mathrm{d}h_{is} - v\mathrm{d}p$. Substituting $v = RT/p$ into eqn. (1.48) and using $C_p = \gamma R/(\gamma - 1)$ gives

$$\frac{\mathrm{d}T}{T} = \frac{(\gamma - 1)}{\gamma\eta_p}\frac{\mathrm{d}p}{p}. \tag{1.49}$$

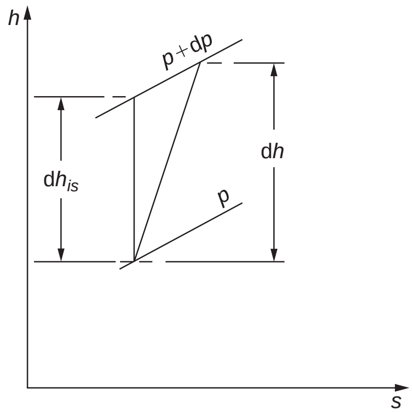


FIGURE 1.11

Incremental Change of State in a Compression Process

Integrating eqn. (1.49) across the whole compressor and taking equal efficiency for each infinitesimal stage gives

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\eta_p\gamma}. \tag{1.50}$$

Now the isentropic efficiency for the whole compression process is

$$\eta_c = (T_{2s} - T_1)/(T_2 - T_1) \tag{1.51}$$

if it is assumed that the velocities at inlet and outlet are equal.

For the *ideal* compression process put $\eta_p = 1$ in eqn. (1.50) and so obtain

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}, \tag{1.52}$$

which is equivalent to eqn. (1.36). Substituting eqns. (1.50) and (1.52) into eqn. (1.51) results in the expression

$$\eta_c = \left[\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} - 1\right] \bigg/ \left[\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\eta_p\gamma} - 1\right]. \tag{1.53}$$

Values of "overall" isentropic efficiency have been calculated using eqn. (1.53) for a range of pressure ratio and different values of η_p; these are plotted in Figure 1.12. This figure amplifies the observation made earlier that the isentropic efficiency of a finite compression process is *less* than the efficiency of the small stages. Comparison of the isentropic efficiency of two machines of different pressure ratios is not a valid procedure since, for equal polytropic efficiency, the compressor with the higher pressure ratio is penalised by the *hidden* thermodynamic effect.

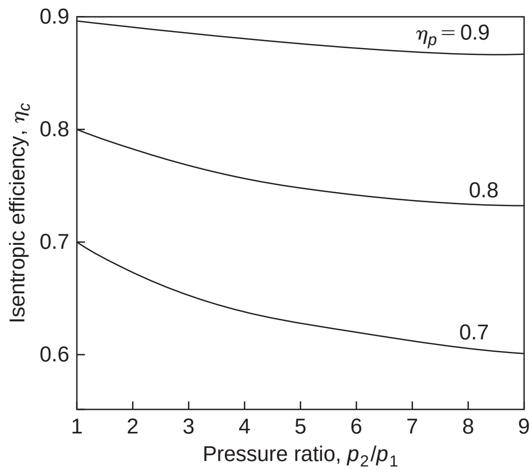


FIGURE 1.12

Relationship Between Isentropic (Overall) Efficiency, Pressure Ratio, and Small Stage (Polytropic) Efficiency for a Compressor ($\gamma = 1.4$)

Example 1.1

An axial flow air compressor is designed to provide an overall total-to-total pressure ratio of 8 to 1. At inlet and outlet the stagnation temperatures are 300 K and 586.4 K, respectively.

Determine the overall total-to-total efficiency and the polytropic efficiency for the compressor. Assume that γ for air is 1.4.

Solution

From eqn. (1.46), substituting $h = C_p T$, the efficiency can be written as

$$\eta_C = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = \frac{\left(\dfrac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1}{T_{02}/T_{01} - 1} = \frac{8^{1/3.5} - 1}{586 \times 4/300 - 1} = 0.85.$$

From eqn. (1.50), taking logs of both sides and rearranging, we get

$$\eta_p = \frac{\gamma - 1}{\gamma}\,\frac{\ln(p_{02}/p_{01})}{\ln(T_{02}/T_{01})} = \frac{1}{3.5} \times \frac{\ln 8}{\ln 1.9547} = 0.8865.$$

Turbine Polytropic Efficiency

A similar analysis to the compression process can be applied to a perfect gas expanding through an adiabatic turbine. For the turbine the appropriate expressions for an expansion, from a state 1 to a state 2, are

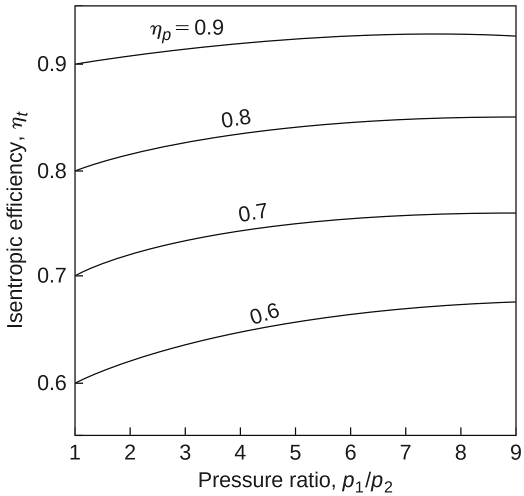


FIGURE 1.13

Turbine Isentropic Efficiency against Pressure Ratio for Various Polytropic Efficiencies ($\gamma = 1.4$)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\eta_p(\gamma-1)/\gamma}, \tag{1.54}$$

$$\eta_t = \left[1 - \left(\frac{p_2}{p_1}\right)^{\eta_p(\gamma-1)/\gamma}\right] \Big/ \left[1 - \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}\right]. \tag{1.55}$$

The derivation of these expressions is left as an exercise for the student. "Overall" isentropic efficiencies have been calculated for a range of pressure ratios and polytropic efficiencies, and these are shown in Figure 1.13. The most notable feature of these results is that, in contrast with a compression process, for an expansion, isentropic efficiency *exceeds* small stage efficiency.

Reheat Factor

The foregoing relations cannot be applied to steam turbines as vapours do not obey the perfect gas laws. It is customary in steam turbine practice to use a *reheat factor* R_H as a measure of the inefficiency of the complete expansion. Referring to Figure 1.14, the expansion process through an adiabatic turbine from state 1 to state 2 is shown on a Mollier diagram, split into a number of small stages. The reheat factor is defined as

$$R_H = \left[(h_1 - h_{xs}) + (h_x - h_{ys}) + \cdots\right]/(h_1 - h_{2s}) = (\Sigma\Delta h_{is})/(h_1 - h_{2s}).$$

Due to the gradual divergence of the constant pressure lines on a Mollier chart, R_H is always greater than unity. The actual value of R_H for a large number of stages will depend upon the position of the expansion line on the Mollier chart and the overall pressure ratio of the expansion. In normal steam turbine practice the value of R_H is usually between 1.03 and 1.08.

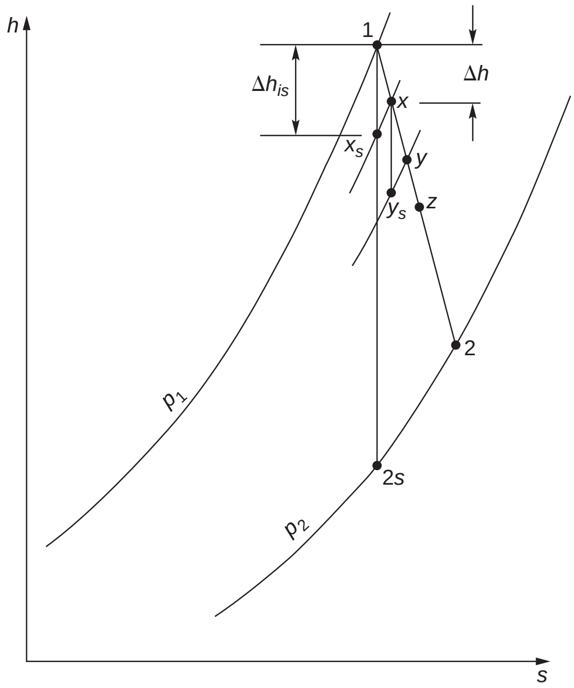


FIGURE 1.14

Mollier Diagram Showing Expansion Process Through a Turbine Split up into a Number of Small Stages

Now since the isentropic efficiency of the turbine is

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{h_1 - h_2}{\Sigma \Delta h_{is}} \times \frac{\Sigma \Delta h_{is}}{h_1 - h_{2s}},$$

then

$$\eta_t = \eta_p R_H, \tag{1.56}$$

which establishes the connection between polytropic efficiency, reheat factor and turbine isentropic efficiency.

1.12 THE INHERENT UNSTEADINESS OF THE FLOW WITHIN TURBOMACHINES

It is a less well-known fact often ignored by designers of turbomachinery that turbomachines can only work the way they do because of *flow unsteadiness*. This subject was discussed by Dean (1959), Horlock and Daneshyar (1970), and Greitzer (1986). Here, only a brief introduction to an extensive subject is given.

In the absence of viscosity, the equation for the stagnation enthalpy change of a fluid particle moving through a turbomachine is

$$\frac{Dh_0}{Dt} = \frac{1}{\rho}\frac{\partial p}{\partial t}, \tag{1.57}$$

where D/Dt is the rate of change following the fluid particle. Eqn. (1.57) shows us that any change in stagnation enthalpy of the fluid is a result of unsteady variations in static pressure. In fact, without unsteadiness, no change in stagnation enthalpy is possible and thus no work can be done by the fluid. This is the so-called "*Unsteadiness Paradox*." Steady approaches can be used to determine the work transfer in a turbomachine, yet the underlying mechanism is fundamentally unsteady.

A physical situation considered by Greitzer is the axial compressor rotor as depicted in Figure 1.15a. The pressure field associated with the blades is such that the pressure increases from the suction surface (S) to the pressure surface (P). This pressure field moves with the blades and is therefore steady in the relative frame of reference. However, for an observer situated at the point* (in the absolute frame of reference), a pressure that varies with time would be recorded, as shown in Figure 1.15b. This unsteady pressure variation is directly related to the blade pressure field via the rotational speed of the blades,

$$\frac{\partial p}{\partial t} = \Omega\frac{\partial p}{\partial \theta} = U\frac{\partial p}{r\partial \theta}. \tag{1.58}$$

Thus, the fluid particles passing through the rotor experience a positive pressure increase with time (i.e., $\partial p/\partial t > 0$) and their stagnation enthalpy is increased.

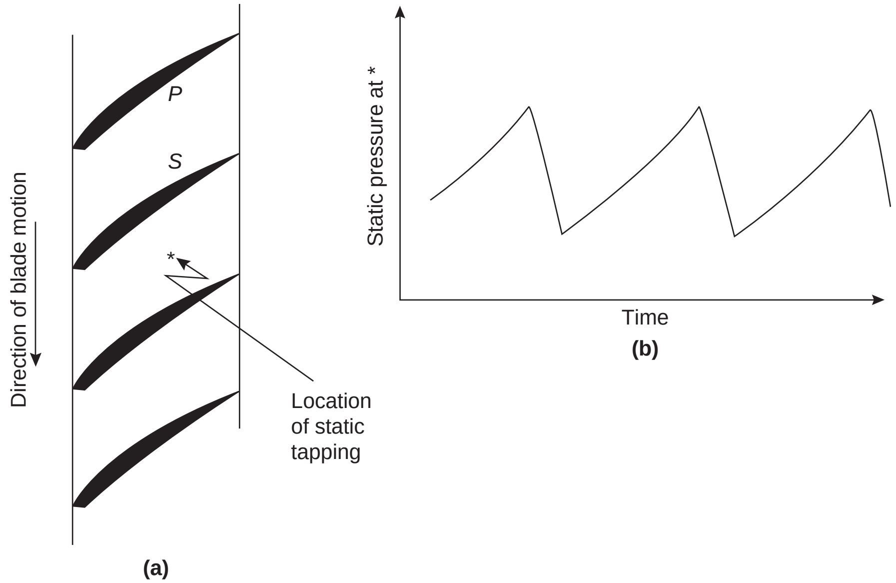


FIGURE 1.15

Measuring the Unsteady Pressure Field of an Axial Compressor Rotor: (a) Pressure Measured at Point* on the Casing, (b) Fluctuating Pressure Measured at Point*

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PROBLEMS

1. For the adiabatic expansion of a perfect gas through a turbine, show that the overall efficiency η_t and small stage efficiency η_p are related by

$$\eta_t = (1-\varepsilon^{\eta_p})/(1-\varepsilon),$$

where $\varepsilon = r^{(1-\gamma)/\gamma}$, and r is the expansion pressure ratio, γ is the ratio of specific heats. An axial flow turbine has a small stage efficiency of 86%, an overall pressure ratio of 4.5 to 1 and a mean value of γ equal to 1.333. Calculate the overall turbine efficiency.

2. Air is expanded in a multi stage axial flow turbine, the pressure drop across each stage being very small. Assuming that air behaves as a perfect gas with ratio of specific heats γ, derive pressure–temperature relationships for the following processes:

(i) reversible adiabatic expansion;
(ii) irreversible adiabatic expansion, with small stage efficiency η_p;
(iii) reversible expansion in which the heat loss in each stage is a constant fraction k of the enthalpy drop in that stage;
(iv) reversible expansion in which the heat loss is proportional to the absolute temperature T.

Sketch the first three processes on a T, s diagram. If the entry temperature is 1100 K and the pressure ratio across the turbine is 6 to 1, calculate the exhaust temperatures in each of the first three cases. Assume that γ is 1.333, that $\eta_p = 0.85$, and that $k = 0.1$.

3. A multistage high-pressure steam turbine is supplied with steam at a stagnation pressure of 7 MPa and a stagnation temperature of 500°C. The corresponding specific enthalpy is

3410 kJ/kg. The steam exhausts from the turbine at a stagnation pressure of 0.7 MPa, the steam having been in a superheated condition throughout the expansion. It can be assumed that the steam behaves like a perfect gas over the range of the expansion and that $\gamma = 1.3$. Given that the turbine flow process has a small-stage efficiency of 0.82, determine

(i) the temperature and specific volume at the end of the expansion,
(ii) the reheat factor.

The specific volume of superheated steam is represented by $pv = 0.231(h = 1943)$, where p is in kPa, v is in m^3/kg, and h is in kJ/kg.

4. A 20 MW back-pressure turbine receives steam at 4 MPa and 300°C, exhausting from the last stage at 0.35 MPa. The stage efficiency is 0.85, reheat factor 1.04, and external losses 2% of the actual isentropic enthalpy drop. Determine the rate of steam flow. At the exit from the first stage nozzles, the steam velocity is 244 m/s, specific volume 68.6 dm^3/kg, mean diameter 762 mm, and steam exit angle 76° measured from the axial direction. Determine the nozzle exit height of this stage.

5. Steam is supplied to the first stage of a five stage pressure-compounded steam turbine at a stagnation pressure of 1.5 MPa and a stagnation temperature of 350°C. The steam leaves the last stage at a stagnation pressure of 7.0 kPa with a corresponding dryness fraction of 0.95. By using a Mollier chart for steam and assuming that the stagnation state point locus is a straight line joining the initial and final states, determine

(i) the stagnation conditions between each stage assuming that each stage does the same amount of work;
(ii) the total-to-total efficiency of each stage;
(iii) the overall total-to-total efficiency and total-to-static efficiency assuming the steam enters the condenser with a velocity of 200 m/s;
(iv) the reheat factor based upon stagnation conditions.